

August 2022 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 \times 10^{-34}$ Js, $\hbar = 1.054571628 \times 10^{-34}$ Js

Boltzmann constant: $k_B = 1.3806504 \times 10^{-23}$ J/K

Elementary charge: $q_e = 1.602176487 \times 10^{-19}$ C

Avogadro number: $N_A = 6.02214179 \times 10^{23}$ particles/mol

Speed of light: $c = 2.99792458 \times 10^8$ m/s

Electron rest mass: $m_e = 9.10938215 \times 10^{-31}$ kg

Proton rest mass: $m_p = 1.672621637 \times 10^{-27}$ kg

Neutron rest mass: $m_n = 1.674927211 \times 10^{-27}$ kg

Bohr radius: $a_0 = 5.2917720859 \times 10^{-11}$ m

Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 \times 10^{-12}$ m

Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ N/A²

Permittivity of free space: $\epsilon_0 = 1/\mu_0 c^2$

Gravitational constant: $G = 6.67428 \times 10^{-11}$ m³/(kg s²)

Stefan-Boltzmann constant: $\sigma = 5.670400 \times 10^{-8}$ W m⁻² K⁻⁴

Wien displacement law constant: $\sigma_w = 2.8977685 \times 10^{-3}$ m K

Planck radiation law: $I(\lambda, T) = (2hc^2/\lambda^5)[\exp(hc/(kT\lambda)) - 1]^{-1}$

Useful integrals:

$$\int \frac{dx}{\sqrt{(x^2 + a^2)^3}} = \frac{x}{a^2(\sqrt{x^2 + a^2})}$$

$$\int \sin mx \sin nx \, dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)}, \quad [m^2 \neq n^2]$$