Equation Aid: The Shroedinger Equation

The Shroedinger equation for a particle with mass m moving in one dimension has the form

$$\frac{-\hbar}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + U(x)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

For a free particle with mass m, such as an electron moving freely through space, U(x) = 0, and the Schroedinger equation is of the form

$$\frac{-\hbar}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

Aside: Solutions to the wave equation for waves on a string

For the wave equation for a wave on a string,

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$$

we have found two linearly independent solution,

$y(x,t) = Acos(kx - \omega t)$ and $y(x,t) = Asin(kx - \omega t)$

with $|v| = \frac{\omega}{k}$
Any other solution is a linear combination of those solutions. The functions

$y(x,t) = e^{i(kx-\omega t)}$ and $y(x,t) = e^{-i(kx-\omega t)}$

are linear combinations of $cos(kx - \omega t)$ and $sin(kx - \omega t)$.

$$e^{i(kx-\omega t)} = cos(kx - \omega t) + isin(kx - \omega t)$$

$$e^{-i(kx-\omega t)} = cos(kx - \omega t) - isin(kx - \omega t)$$