Equation Aid: Lab 3 Complex Numbers

The derivative of $cos(\psi) + isin(\psi)$

Since $\frac{d\,cos(\psi)}{d\psi} = -sin(\psi)$ and $\frac{d\,sin(\psi)}{d\psi} = cos(\psi)$ we get

$$\frac{d\,cos(\psi) + isin(\psi)}{d\psi} = -sin(\psi) + icos(\psi)$$

Since $(-1) = i^2$, this can be written

$$\frac{d\,cos(\psi) + isin(\psi)}{d\psi} = i^2 sin(\psi) + icos(\psi) = i(cos(\psi) + isin(\psi))$$

To express this result more formally, let us write $f(\psi) = cos(\psi) + isin(\psi)$. Then

$$\frac{d\,f(\psi)}{d\psi} = if(\psi)$$

To within a constant i, the function $f(\psi)$ is equal to its own derivative. What function, that you are already familiar with, behaves this way? The exponential function!

Recall that $\frac{d\,e^{ax}}{dx} = ae^{ax}$.

Thus if we replace x by ψ and a by i , we get

$$\frac{d\,e^{i\psi}}{d\psi} = ie^{i\psi}$$

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Division:

Let $z = \frac{z_1}{z_2} = \frac{z_1 z_2^*}{z_2 z_2^*} = \frac{z_1 z_2^*}{|z_2|^2}$. We make the denominator real.

$$z = \frac{(x_1 + iy_1)(x_2 - iy_2)}{x_2^2 + y_2^2} = \frac{x_1x_2 - ix_1y_2 + iy_1x_2 + y_1y_2}{x_2^2 + y_2^2} = \frac{(x_1x_2 + y_1y_2) + i(-x_1y_2 + y_1x_2)}{x_2^2 + y_2^2}$$

If $z = x + iy$, then $x = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2}$ and $y = \frac{-x_1y_2 + y_1x_2}{x_2^2 + y_2^2}$. Division mixes the real and imaginary parts of the two numbers.