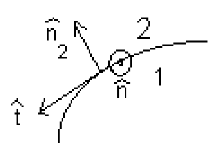
Formulas:

Mechanics

Motion in a non-inertial frame:	$m d\mathbf{v}/dt = -\partial U/\partial \mathbf{r} - m d\mathbf{V}/dt + m\mathbf{r} \times d\mathbf{\Omega}/dt - 2m\mathbf{\Omega} \times \mathbf{v} - m\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})$
Lagrange's equations:	$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_j}\right) - \frac{\partial L}{\partial q_j} = 0$, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$, $Q_j = -\frac{\partial U}{\partial q_j}$ or $Q_j = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\left(\frac{\partial U}{\partial \dot q_j}\right)$
Hamilton's equations:	$\dot q_i = \frac{\partial H}{\partial p_i}$, $\dot p_i = -\frac{\partial H}{\partial q_i}$, $H(q,p,t) = \sum_i p_i \dot q_i - L$, $\frac{\partial L}{\partial \dot q_j} = p_j$
Lagrange multipliers:	$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_k}\right) - \frac{\partial L}{\partial q_k} = \sum_l \lambda_l a_{lk}$, $\sum_k a_{lk} dq_k + a_{lt} dt = 0, \quad l = 1, \cdots, m$
Small oscillations:	$L = \frac{1}{2}\sum_{ij}\left(T_{ij}\dot q_i \dot q_j - k_{ij} q_i q_j\right)$ with $T_{ij} = T_{ji}$, $k_{ij} = k_{ji}$ $\sum_j \left(k_{ij} - \omega_\alpha^2 T_{ij}\right) A_{j\alpha} = 0$ $q_j = \mathrm{Re} \sum_\alpha \left(C_\alpha A_{j\alpha} e^{i\omega_\alpha t}\right)$
Motion in a central potential:	$m\ddot r - \frac{M^2}{mr^3} = f(r)$, or $m\ddot r = -\frac{\partial U_{eff}(r)}{\partial r}$, with $f(r) = -\frac{\partial U(r)}{\partial r}$ $\frac{M^2 u^2}{m}\left(\frac{d^2 u}{d\phi^2} + u\right) = -f(u)$ $\dot r = \pm\sqrt{\frac{2}{m}\left(E - U_{eff}(r)\right)}$, $\phi = \phi_0 - \int \frac{M du}{\sqrt{2m\left(E - U_{eff}(u)\right)}}$ Kepler orbit: $\frac{p}{r} = 1 + e\cos\left(\phi - \phi_0\right)$
Two interacting particles:	CM frame: $L = \frac{1}{2}\mu \dot{\mathbf{r}}^2 - U(r)$, $\mu = \frac{m_1 m_2}{m_1 + m_2}$, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$
Relativistic kinematics:	$\begin{pmatrix} x_0' \\ x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0' \\ x_1' \\ x_2' \\ x_3' \end{pmatrix}$

4-vectors:	$(u_0, \boldsymbol{u}) = \left(\gamma c, \gamma\frac{d\boldsymbol{r}}{dt}\right) = \left(\frac{dx_0}{d\tau}; \frac{d\boldsymbol{r}}{d\tau}\right)$, $(u_0, \boldsymbol{u}) \cdot (u_0, \boldsymbol{u}) = c^2$ $\left(p_0, \boldsymbol{p}\right) = (mu_0, m\boldsymbol{u}) = \left(\gamma mc, \gamma m\frac{d\boldsymbol{r}}{dt}\right)$, $p_0 = \frac{E}{c} = \gamma mc$, $\boldsymbol{p} = \gamma m\frac{d\boldsymbol{r}}{dt}$ $\left(p_0, \boldsymbol{p}\right) \cdot \left(p_0, \boldsymbol{p}\right) = m^2c^2 \Rightarrow E^2 = m^2c^4 + p^2c^2$
Transformation of velocities:	$u'_{\parallel} = (u_{\parallel} - v)/(1 - \mathbf{v}\cdot\mathbf{u}/c^2)$, $u'_{\perp} = u_{\perp}/(\gamma(1 - \mathbf{v}\cdot\mathbf{u}/c^2))$
Doppler shift:	$\omega' = \gamma\omega(1 - (v/c)\cos\theta)$
Relativistic collisions:	For each component p_μ of the 4-vector (p_0, p_1, p_2, p_3) we have $\sum_{particles\_in} p_\mu = \sum_{particles\_out} p_\mu$, or $\sum_i \left(p_i\right)_\mu = \sum_j \left(p_j\right)_\mu$. For transformations between reference frames we have $(P_0, \boldsymbol{P}) \cdot (P_0, \boldsymbol{P}) = \left(P'_0, \boldsymbol{P}'\right) \cdot \left(P'_0, \boldsymbol{P}'\right)$, where $P_0 = \sum_{particles} p_0$ and $\boldsymbol{P} = \sum_{particles} \boldsymbol{p}$.

E&M

Maxwell's equations:	$\vec{\nabla}\cdot\boldsymbol{E}=\frac{\rho}{\varepsilon_0},\quad \vec{\nabla}\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t}\qquad \vec{\nabla}\cdot\boldsymbol{B}=0,\quad \vec{\nabla}\times\boldsymbol{B}=\mu_0\boldsymbol{j}+\frac{1}{c^2}\frac{\partial\boldsymbol{E}}{\partial t}$ $\vec{\nabla}\cdot\boldsymbol{D}=\rho_f,\quad \vec{\nabla}\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t}\qquad \vec{\nabla}\cdot\boldsymbol{B}=0,\quad \vec{\nabla}\times\boldsymbol{H}=\boldsymbol{j}_f+\frac{\partial\boldsymbol{D}}{\partial t}$ lih materials: $\vec{\nabla}\cdot\boldsymbol{E}=\frac{\rho_f}{\varepsilon},\quad \vec{\nabla}\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t},$ $\vec{\nabla}\cdot\boldsymbol{B}=0,\quad \vec{\nabla}\times\boldsymbol{B}=\mu\boldsymbol{j}_f+\mu\varepsilon\frac{\partial\boldsymbol{E}}{\partial t}$
Dipole field and potential:	$\boldsymbol{E}(\boldsymbol{r})=\left[\frac{1}{4\pi\varepsilon_0}\right]_{SI}\frac{1}{r^3}\left[3\left(\boldsymbol{p}\cdot\hat{\boldsymbol{r}}\right)\hat{\boldsymbol{r}}-\boldsymbol{p}\right]\qquad \phi(\boldsymbol{r})=\left[\frac{1}{4\pi\varepsilon_0}\right]_{SI}\frac{\boldsymbol{p}\cdot\hat{\boldsymbol{r}}}{r^2}$ **F** = ∇(**p**·**E**), **τ** = **p**×**E**, U = -(**p**·**E**) $\boldsymbol{m}=IA\hat{\boldsymbol{n}}=\frac{1}{2}\int_V\boldsymbol{r}\times\boldsymbol{j}(\boldsymbol{r})dV$ $\boldsymbol{B}(\boldsymbol{r})=\frac{\mu_0}{4\pi}\frac{3(\boldsymbol{m}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}-\boldsymbol{m}}{r^3},\quad \boldsymbol{A}(\boldsymbol{r})=\frac{\mu_0}{4\pi}\frac{\boldsymbol{m}\times\hat{\boldsymbol{r}}}{r^2},$ $\boldsymbol{F}=\vec{\nabla}(\boldsymbol{m}\cdot\boldsymbol{B})\ ,\quad \boldsymbol{\tau}=\boldsymbol{m}\times\boldsymbol{B}$
Boundary conditions:	$(\boldsymbol{E}_2-\boldsymbol{E}_1)\cdot\hat{\boldsymbol{n}}_2=\frac{\sigma}{\varepsilon_0}\ ,\ (\boldsymbol{E}_2-\boldsymbol{E}_1)\cdot\hat{\boldsymbol{t}}=0\ ,\ (\boldsymbol{D}_2-\boldsymbol{D}_1)\cdot\hat{\boldsymbol{n}}_2=\sigma_f$ $(\boldsymbol{B}_2-\boldsymbol{B}_1)\cdot\hat{\boldsymbol{n}}_2=0\ ,\ (\boldsymbol{B}_2-\boldsymbol{B}_1)\cdot\hat{\boldsymbol{t}}=\mu_0\boldsymbol{k}\cdot\hat{\boldsymbol{n}}\ ,\ (\boldsymbol{H}_2-\boldsymbol{H}_1)\cdot\hat{\boldsymbol{t}}=\boldsymbol{k}_f\cdot\hat{\boldsymbol{n}}$  Note: **t** × **n**$_2$ = -**n**
Multipole expansion:	$\phi(r)=\left[\frac{1}{4\pi\varepsilon_0}\right]_{SI}\left\|\frac{1}{r}\int_{V'}\rho(\boldsymbol{r}')dV'+\frac{1}{r^2}\hat{\boldsymbol{r}}\cdot\int_{V'}\rho(\boldsymbol{r}')\boldsymbol{r}'dV'\right.$ $\left.+\frac{1}{r^3}\int_{V'}\frac{3(\hat{\boldsymbol{r}}\cdot\boldsymbol{r}')^2-\boldsymbol{r}'^2}{2}\rho(\boldsymbol{r}')dV'+\cdots\right]$ $=\left[\frac{1}{4\pi\varepsilon_0}\right]_{SI}\left[\frac{Q}{r}+\frac{\boldsymbol{p}\cdot\hat{\boldsymbol{r}}}{r^2}+\frac{1}{2r^5}\sum_{ij}3x_ix_jQ_{ij}+\cdots\right]$ $Q_{ij}=\int\left(x_i'x_j'-\frac{1}{3}\delta_{ij}r'^2\right)\rho(\boldsymbol{r}')dV',\quad Q_{ji}=Q_{ij},\quad \sum_i Q_{ii}=0$ $W=q\phi(0)-\boldsymbol{p}\cdot\boldsymbol{E}(0)-\frac{1}{2}\sum_i\sum_j Q_{ij}\frac{\partial E_j}{\partial x_i}\Big\|_{x_i=0}+\cdots$
Method of images:	Grounded conducting sphere: place $q'=-q\frac{R}{d}$ at $d'=\frac{R^2}{d}$ to make the sphere an equipotential with φ = 0.

Boundary value problems:	$\phi(r,\theta) = \sum_{n=0}^{\infty}(A_n r^n + B_n r^{-(n+1)})P_n(\cos\theta)$
Dielectrics:	$\rho = \rho_f + \rho_p$, $\sigma = \sigma_f + \sigma_p$, $\rho_p = -\vec{\nabla}\cdot\boldsymbol{P}$, $\sigma_p = \boldsymbol{P}\cdot\hat{\boldsymbol{n}}$ $\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P}$ lih dielectrics: $\boldsymbol{P} = \varepsilon_0\chi_e\boldsymbol{E}$, $\boldsymbol{D} = \varepsilon_0(1+\chi_e)\boldsymbol{E} = \varepsilon_0 k_e\boldsymbol{E} = \varepsilon\boldsymbol{E}$, $\vec{\nabla}\cdot\boldsymbol{D} = \rho_f$, $\nabla^2\phi = -\frac{\rho_f}{\varepsilon}$
Magnetic materials:	$\boldsymbol{j} = \boldsymbol{j}_f + \boldsymbol{j}_m$, $\boldsymbol{k} = \boldsymbol{k}_f + \boldsymbol{k}_m$, $\boldsymbol{j}_m = \vec{\nabla}\times\boldsymbol{M}$, $\boldsymbol{k}_m = \boldsymbol{M}\times\hat{\boldsymbol{n}}$, $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$ lih magnetic materials: $\boldsymbol{M} = \chi_m\boldsymbol{H}$, $\boldsymbol{B} = \mu_0(\boldsymbol{H}+\boldsymbol{M}) = \mu_0(1+\chi_m)\boldsymbol{H} = \mu_0 k_m\boldsymbol{H} = \mu\boldsymbol{H}$, $\vec{\nabla}\times\boldsymbol{H} = \boldsymbol{j}_f$
Quasi-static situations:	$\varepsilon_i = -\sum_j M_{ij}\frac{\partial I_j}{\partial t}$ $\varepsilon = -L\frac{\partial I}{\partial t}$ $U = \frac{1}{2}\sum_{m=1}^{N}F_m I_m$, $U = \frac{1}{2\mu_0}\int_{all\_space}\boldsymbol{B}\cdot\boldsymbol{B}dV$
Electrodynamics:	$\vec{\nabla}\times\left(\boldsymbol{E}+\frac{\partial\boldsymbol{A}}{\partial t}\right) = 0 \Rightarrow \boldsymbol{E}+\frac{\partial\boldsymbol{A}}{\partial t} = -\vec{\nabla}\phi$
Poynting's theorem:	$\boldsymbol{E}\cdot\boldsymbol{j} = -\frac{\partial u}{\partial t} - \vec{\nabla}\cdot\boldsymbol{S}$ $u = \frac{1}{2\mu_0}B^2 + \frac{\varepsilon_0}{2}E^2$ $\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B}$
The Lorentz gauge:	$\vec{\nabla}\cdot\boldsymbol{A} = -\frac{1}{c^2}\frac{\partial\phi}{\partial t}$
The wave equation:	$\left(\nabla^2 - \mu\varepsilon\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E} = 0$ $\left(\nabla^2 - \mu\varepsilon\frac{\partial^2}{\partial t^2}\right)\boldsymbol{B} = 0$ $\left(\nabla^2 - \mu\varepsilon\frac{\partial^2}{\partial t^2} - \mu\sigma_c\frac{\partial}{\partial t}\right)\boldsymbol{E} = 0$, $\left(\nabla^2 - \mu\varepsilon\frac{\partial^2}{\partial t^2} - \mu\sigma_c\frac{\partial}{\partial t}\right)\boldsymbol{B} = 0$, $k^2 = i\mu\sigma_c\omega + \mu\varepsilon\omega^2 = \mu\varepsilon(\omega)\omega^2$
Electromagnetic radiation:	$\boldsymbol{E}(\boldsymbol{r},t) = -\left[\frac{1}{4\pi\varepsilon_0}\right]_{SI}\frac{q}{c^2 r''}\boldsymbol{a}_\perp(t-\frac{r''}{c})$, $\boldsymbol{r}'' = \boldsymbol{r} - \boldsymbol{r}'(t-\frac{\lvert\boldsymbol{r}-\boldsymbol{r}'\rvert}{c})$, $\boldsymbol{B} = \frac{\hat{\boldsymbol{r}}''}{c}\times\boldsymbol{E}$
Larmor formula:	$P = \oint\vec{S}\cdot\hat{n}dA = \frac{2}{3}\frac{e^2a^2}{c^3}$ = $q^2a^2/(6\pi\varepsilon_0c^3)$
4-vectors:	$j^\mu = (c\rho,\mathbf{j})$ = 4-vector current, $A^\mu = (\phi/c,\mathbf{A})$ = 4-vector potential
Transformation of the fields:	$\mathbf{E}'_\parallel = \mathbf{E}_\parallel$, $\mathbf{B}'_\parallel = \mathbf{B}_\parallel$, $\mathbf{E}'_\perp = \gamma(\mathbf{E} + \mathbf{v}\times\mathbf{B})_\perp$, $\mathbf{B}'_\perp = \gamma(\mathbf{B} - (\mathbf{v}/c^2)\times\mathbf{E})_\perp$
Lorentz invariants:	$\mathbf{E}^2 - c^2\mathbf{B}^2$, $(\mathbf{E}\cdot\mathbf{B})^2$

Quantum Mechanics

WKB approximation:	$\oint\ p\,dx = \oint\ \hbar k\,dx = (n + 1/2)h$, $k^2 = (2m/\hbar^2)(E - V(x))$, V(x) finite everywhere. In regions where E > V(x) we have $\phi(x) = Ak^{-1/2}\exp(\pm i\int^x k(x')dx')$, and in regions where E < V(x) we have $\phi(x) = A\rho^{-1/2}\exp(\pm\int^x \rho k(x')dx')$.
Harmonic oscillator:	$\phi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left(-\frac{1}{2}\frac{m\omega x^2}{\hbar}\right)$ $\phi_1(x) = \left(\frac{4}{\pi}\left(\frac{m\omega}{\hbar}\right)^3\right)^{\frac{1}{4}} x\exp\left(-\frac{1}{2}\frac{m\omega x^2}{\hbar}\right)$ $\phi_2(x) = \left(\frac{m\omega}{4\pi\hbar}\right)^{\frac{1}{4}} \left[2\frac{m\omega}{\hbar}x^2 - 1\right] \exp\left(-\frac{1}{2}\frac{m\omega x^2}{\hbar}\right)$ $\phi_n(x) = \frac{1}{\sqrt{n!}}\frac{1}{\sqrt{2^n}}\left(\frac{\beta}{\sqrt{\pi}}\right)^{\frac{1}{2}} H_n(\eta)\exp\left(-\frac{1}{2}\eta^2\right)$, where $\eta = \sqrt{\frac{m\omega}{\hbar}}\,x = \beta x$
Angular momentum:	$[J_i,J_j] = \varepsilon_{ijk} i\hbar J_k$, $[J_i,J^2] = 0$, $J^2\|k,j,m\rangle = j(j+1)\hbar^2\|k,j,m\rangle$, $J_z\|k,j,m\rangle = m\hbar\|k,j,m\rangle$, $J_+ = J_x + iJ_y$, $J_+ = J_x - iJ_y$, $J_\pm\|k,j,m\rangle = [j(j+1)-m(m\pm1)]^{1/2}\hbar\|k,j,m\pm1\rangle$.
Orbital angular momentum:	$L^2 = -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)$ $\quad L_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$ $Y_{00} = \frac{1}{\sqrt{4\pi}}$, $\quad Y_{1\pm1}(\theta,\phi) = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}$, $\quad Y_{10}(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta$ $Y_{2\pm2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\phi}$, $Y_{2\pm1}(\theta,\phi) = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\phi}$, $Y_{20}(\theta,\phi) = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$
Spin 1/2:	$(S_z) = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\quad (S_x) = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $(S_y) = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\quad (S^2) = \frac{3\hbar^2}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Particle in a central potential:	$H = -(\hbar^2/(2m))(1/r)(\partial^2/\partial r^2)r + L^2/(2mr^2) + V(r)$, $\psi_{klm}(r,\theta,\phi) = R_{kl}(r)Y_{lm}(\theta,\phi) = [u_{kl}(r)/r]Y_{lm}(\theta,\phi)$, $[-(\hbar^2/(2m))(\partial^2/\partial r^2) + \hbar^2 l(l+1)/(2mr^2) + V(r)]u_{kl}(r) = E_{kl}u_{kl}(r)$.
Stationary perturbation theory:	$E_1^p = \langle\phi_p \| W \| \phi_p\rangle$, $\|\psi_p^1\rangle = \Sigma_{p'\neq p,i}\, b_{p'}^i \| \phi_{p'}^i\rangle$, where $b_{p'}^i = \langle\phi_{p'}^i \| W \| \phi_p\rangle / (E_0^p - E_0^{p'})$, $E_2^p = \Sigma_{p'\neq p,i}\ \|\langle\phi_{p'}^i \| W \| \phi_p\rangle\|^2 / (E_0^p - E_0^{p'})$.
Time-dependent perturbation theory: Fermi's golden rule:	$P_{if}(t) = (1/\hbar^2)\|\int_0^t \exp(i\omega_{fi}t')W_{fi}(t')dt'\|^2$ Let $W(t) = W\exp(\pm i\omega t)$, then $w(i,\beta E) = (2\pi/\hbar)\rho(\beta,E)\|W_{Ei}\|^2\delta_{E-Ei,\hbar\omega}$, where $W_{Ei} = \langle\phi_E\|W\|\phi_i\rangle$.
Scattering:	Asymptotic form: $\phi_k(r) = \exp(ikz) + f_k(\theta)\exp(ikr)/r$. $f_k(\theta) = (1/k)\Sigma_{l=0}^{\infty}(2l+1)\exp(i\delta_l)\sin\delta_l P_l(\cos\theta)$, $d\sigma_k/d\Omega = \|f_k(\theta)\|^2 = (1/k^2)\|\Sigma_{l=0}^{\infty}(2l+1)\exp(i\delta_l)\sin\delta_l P_l(\cos\theta)\|^2$. $\sigma_k^B(\theta,\phi) = \sigma_k^B(\mathbf{k},\mathbf{k}') = [\mu^2/(4\pi^2\hbar^4)]\|\int d^3r' \exp(-i\mathbf{q}\cdot\mathbf{r}')V(\mathbf{r}')\|^2$, where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$, $\mathbf{k} = \mu\mathbf{v}_0/\hbar$, $\mathbf{k}' = \mu v_0/\hbar\ (\mathbf{k}'/k')$, and is the reduced mass.

Vector identities:
Note: **A**, **B**, **C**, and **D** are vectors.

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}$$

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})$$

Note: **A** and **B** are vector fields, ψ and ϕ are scalar fields.

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla\cdot(\psi\mathbf{A}) = \mathbf{A}\cdot\nabla\psi + \psi\nabla\cdot\mathbf{A}$$

$$\nabla\times(\psi\mathbf{A}) = \psi\nabla\times\mathbf{A} - \mathbf{A}\times\nabla\psi$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}(\nabla\times\mathbf{B})$$

$$\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$$

$$(\mathbf{A}\cdot\nabla)\mathbf{A} = \frac{1}{2}\nabla|\mathbf{A}|^2 + (\nabla\times\mathbf{A})\times\mathbf{A}$$

$$\nabla\times\nabla\phi = 0$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

$$\nabla\cdot\nabla\phi = \nabla^2\phi$$

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

Integral relations:

$$\int_V (\nabla\cdot\mathbf{F})\,dV = \oint_S \mathbf{F}\cdot d\mathbf{a}$$ Divergence theorem

$$\int_S (\nabla\times\mathbf{F})\cdot d\mathbf{a} = \oint_C \mathbf{F}\cdot d\mathbf{s}$$ Stokes' theorem

$$\int_S \phi(\nabla\psi)\cdot d\mathbf{a} = \int_V [\phi\nabla^2\psi + (\nabla\phi)\cdot(\nabla\psi)]\,dV.$$ Green's first identity

$$\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\,dV = \int_S (\phi\nabla\psi - \psi\nabla\phi)\cdot d\mathbf{a}.$$ Green's second identity

Gradient, divergence, curl, and Laplacian:

Note: **A** is a vector fields and *f* is a scalar fields.

Cartesian coordinates:

∇f	=	$\frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$
$\nabla \cdot \mathbf{A}$	=	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
$\nabla \times \mathbf{A}$	=	$(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z})\hat{\mathbf{x}} \quad +$ $(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x})\hat{\mathbf{y}} \quad +$ $(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})\hat{\mathbf{z}}$
$\nabla^2 f$	=	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Cylindrical coordinates:

∇f	=	$\frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$
$\nabla \cdot \mathbf{A}$	=	$\frac{1}{\rho}\frac{\partial \rho A_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$
$\nabla \times \mathbf{A}$	=	$(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z})\hat{\boldsymbol{\rho}} \quad +$ $(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho})\hat{\boldsymbol{\phi}} \quad +$ $\frac{1}{\rho}(\frac{\partial \rho A_\phi}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi})\hat{\boldsymbol{z}}$
$\nabla^2 f$	=	$\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\frac{\partial f}{\partial \rho}) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$

Spherical coordinates:

∇f	=	$\frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\phi}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$
$\nabla \cdot \mathbf{A}$	=	$\frac{1}{r^2}\frac{\partial r^2 A_r}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial A_\theta \sin\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$
$\nabla \times \mathbf{A}$	=	$\frac{1}{r\sin\theta}(\frac{\partial A_\phi \sin\theta}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi})\hat{\boldsymbol{r}} \quad +$ $(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial r A_\phi}{\partial r})\hat{\boldsymbol{\theta}} \quad +$ $\frac{1}{r}(\frac{\partial r A_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta})\hat{\boldsymbol{\phi}}$
$\nabla^2 f$	=	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial f}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial f}{\partial \theta}) + \frac{1}{r^2\sin\theta}\frac{\partial^2 f}{\partial \phi^2}$