January 2016 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\,\lambda)) - 1]^{-1}$

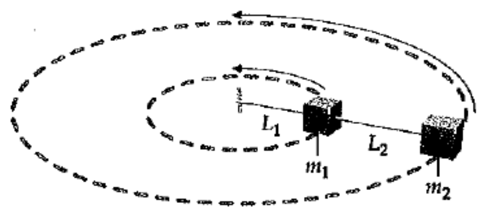
Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:
A satellite moves on a circular earth orbit that has a radius of 6700 km. The radius of Earth is 6371 km. A model airplane is flying on a 15 m guideline in a horizontal circle. The guideline is parallel to the ground. Find the speed of the plane such that the plane and the satellite have the same centripetal acceleration.

Problem 2:
Find the capacitance of two concentric spherical metal shells, with radii a and b, b > a.

Problem 3:
A proton (mass = $1.7*10^{-27}$ kg, charge = $1.6*10^{-19}$ C) is accelerated through a voltage of 500 V and enters a magnetic field perpendicular to its direction of travel. The magnetic field strength is 1.5 Tesla. What is the radius of curvature of the proton's path in the magnetic field?

Problem 4:
A block of mass m_1 is attached to a cord of length L_1. The mass moves in a horizontal circle with no friction. A second block of mass m_2 is attached to the first by a cord of length L_2 and also moves in a circle. If the period of the motion is T, find the tension in each cord in terms of L_1, m_1, L_2, m_2 and T.



Problem 5:
Two gratings A and B have slit separations d_A and d_B, respectively.
They are used with the same light source and the same observation screen. When grating A is replaced with grating B, it is observed that the first-order maximum of A is exactly replaced by the second-order maximum of B.
(a) Determine the ratio d_B/d_A.
(b) Find the next two principal maxima of grating A and the principal maxima of grating B that exactly replace them when the gratings are switched. Identify these maxima by their order numbers.

Problem 6:
A space traveler, moving with velocity v**i** with respect to Earth synchronizes his clock with a friend on Earth at $t' = t = 0$. The earthman then observes both clocks and simultaneously reads the times t and t' (t directly and t' through a telescope). What does t read when t' reads 1 hour?

Problem 7:
Is each of the following sets a valid combination of quantum numbers (n, ℓ, m_ℓ, m_s) for the energy eigenstates of hydrogen? If not, explain why not.
(2, 2, -1, ½)
(3, 1, +2, -½)
(3, 1, 0, ½)
(4, 1, 1, -3/2)
(2, -1, 1, +1/2)

Problem 8:
A ball of mass 100 g hangs on a string that is tightly wrapped around a uniform cylinder that has a mass 500 g and a radius of 5 cm. The ball is released from rest and drops a distance of 7 m to the floor while the cylinder is spinning about its fixed symmetry axis to release the string. What is the speed at which the ball hits the floor?

Problem 9:
Suppose you have two particles, one in single particle states state Φ_1 and Φ_2, respectively. Assuming that Φ_1 and Φ_2 are orthonormal, construct a two-particle wave function assuming that the particles are
(a) distinguishable,
(b) bosons, and
(c) fermions.

Problem 10:
Provide a brief one-sentence description of the collaborative contribution made by each of the following pairs of scientists.
(a) Breit-Wigner
(b) Bose-Einstein
(c) Fermi-Dirac
(d) Hertzsprung- Russell

Section II:
Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:
A particle in a potential well $U(x)$ is initially in a state whose wavefunction is an equal-weight superposition of the ground state and first excited state wavefunctions

$$\Psi(x, t = 0) = C[\psi_1(x) + \psi_2(x)],$$

where C is a constant and $\psi_1(x)$ and $\psi_2(x)$ are normalized solutions to the time-independent Schrödinger equation with energies E_1 and E_2.

(a) Show that the value $C = 1/\sqrt{2}$ normalizes $\Psi(x,0)$.
(b) Determine $\Psi(x,t)$ at any later time t.
(c) Show that the average energy $<E>$ for $\Psi(x, t)$ is the arithmetic mean of the energies E_1 and E_2.
(d) Determine the uncertainty ΔE of the energy for $\Psi(x,t)$.

Problem 12:
An electromagnetic plane wave with wavelength λ propagates in vacuum along the direction indicated by the vector $2\hat{x} + \hat{y}$. The electric field has amplitude E_0, and is oriented along the vertical z direction. Write the expression for the electric and magnetic fields knowing that at the origin, at time t = 0, the electric field points in the negative z-direction with magnitude equal to $0.5\ E_0$, and its magnitude is increasing.

Problem 13:
A proton is fired directly at alpha particle (He^{2+}) such that the two particles are initially approaching one another with the same (non-relativistic) speed v_0 when they are far apart. What is the classical distance of closest approach of the two particles?

Problem 14:
The Lagrangian of a system of N degrees of freedom is

$$L = \frac{1}{2}\sum_{i,j=1}^{N} \dot{q}_i M_{ij} \dot{q}_j \; + \sum_{i=1}^{N} A_i \dot{q}_i \; = \; \frac{1}{2}\vec{\dot{q}}^{T} \overleftrightarrow{M} \; \vec{\dot{q}} + \vec{A}\cdot\vec{\dot{q}} \, .$$

What is the Hamiltonian for a symmetric mass matrix $M_{ij} = M_{ji}$?

Problem 15:
(a) One mole of ideal gas with constant heat capacity C_V is placed inside a cylinder. Inside the cylinder there is a piston which can move without friction along the vertical axis. Pressure P_1 is applied to the piston and the gas temperature is T_1. At some point, P_1 is **abruptly** changed to P_2 (e.g. by adding or removing a weight from the piston). As a result, the gas volume changes adiabatically. Find the temperature T_2 and the volume V_2 after the thermodynamic equilibrium has been reached in terms of C_V, P_1, T_1, and P_2.
Use the relation between heat capacities C_V and C_P to simplify the formulas.

(b) After the thermodynamic equilibrium has been established in part (a), the pressure is abruptly reset to its original value P_1. Compute final values of the temperature T_f and the volume V_f after the thermodynamic equilibrium has been reached again.
Compute the difference in temperatures $(T_f - T_1)$ and show that it is quadratic in $(P_2 - P_1)$.
Comment on both the sign of the temperature difference.