January 2014 Qualifying Exam

Part II

Mathematical tables are allowed. Formula sheets are provided.

Calculators are allowed.

Please clearly mark the problems you have solved and want to be graded.
Do only mark the required number of problems.

Physical Constants:

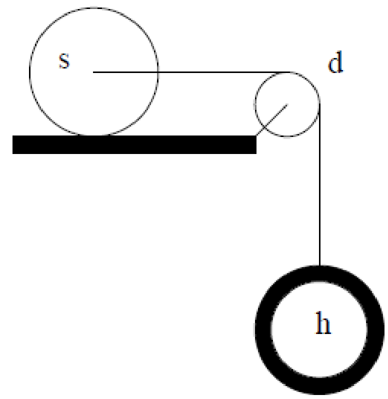
Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

Solve 6 out of the 8 problems! (All problems carry the same weight)

Problem 1:

A hoop with mass m_h = 2.7 kg and radius R_h = 0.15 m hangs from a string that goes over a solid disk pulley with mass m_d = 2.1 kg and radius R_d = 0.09 m. The other end of the string is attached to a massless axel through the center of a sphere on a flat horizontal surface that rolls without slipping and has mass m_s = 3.7 kg and radius R_s = 0.2 m. The system is released from rest.

(a) What is the magnitude of the linear acceleration of the hoop?
(b) What is the tension provided by the string that holds the hoop?
(c) What is the angular acceleration of the disk pulley?



Problem 2:

A cylindrical bar magnet of length L and radius R has magnetization **M** directed parallel to its sides. Compute the magnetic field **B** on the symmetry axes of the magnet at a distance z > L from the center of the magnet.

Problem 3:

The spin-dependent Hamiltonian of an electron-positron system in the presence of a uniform magnetic field B in z-direction can be written as

$$H = A\mathbf{S}^{(e-)}\cdot\mathbf{S}^{(e+)} + (eB/m)(S_z^{(e-)} - S_z^{(e+)}),$$

where A is a positive constant. Find the lowest energy eigenvalue for this system.

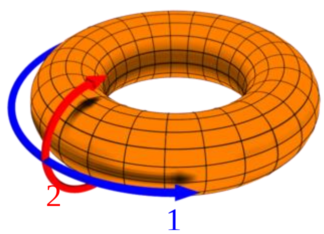
Problem 4:

In the film 2001: A Space Odyssey, a toroidal space station rotates about a fixed axis providing a centrifugal acceleration equal to the Earth's gravitational acceleration of $g = 10$ m/s^2 on a stationary object located at the outer radius ("floor") of the space station.

(a) Given that the outer radius is 150 m, find the needed rotational frequency ω of the space station.

(b) Find the (fictitious) acceleration that would be felt by a person walking at 1.3 m/s for each of the two orthogonal directions, respectively, along the "floor" (see drawing).

(c) Find the (fictitious) acceleration that would be felt be a person sitting down or rising at 1.3 m/s from a chair on the floor of the space station.



Problem 5:

In a reference frame S the 4-vector potential $A^\mu = (\Phi/c, \mathbf{A})$ is given as

$$A^\mu = (-Ey/c, -By/2, Bx/2, 0).$$

Reference frame S' moves along the x direction with speed v = E/B.

Note E and B are constants and $B \neq 0$.

(a) Carry out the Lorentz transformation of the 4-vector potential from S to S'.

(b) Find the electric and magnetic fields before and after the Lorentz transformation.

Problem 6:
A quantum-mechanical particle of mass m in a simple harmonic oscillator potential $V(x) = \frac{1}{2}kx^2$ has energy eigenvalues $E_n = (n + 1/2)\hbar\omega$, $n = 0, 1, 2, ...$, where $\omega = \sqrt{k/m}$,

and the normalized wave functions $\psi_n(x) = \left(2^n n!\right)^{-1/2} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(\frac{-\xi^2}{2}\right) H_n(\xi)$, where

$\xi = \sqrt{\frac{m\omega}{\hbar}}\, x$ and $H_n(\xi) = (-1)^n e^{\xi^2} \frac{\partial^n}{\partial \xi^n} e^{-\xi^2}$.

(a) What are the energy eigenvalues E'_n and normalized wave functions $\psi'_n(x)$ of a quantum-mechanical particle of mass m in a potential $V'(x) = \begin{cases} \infty, & x \le 0 \\ \frac{1}{2}kx^2, & x > 0 \end{cases}$?

(b) Evaluate <x> for the ground state of this potential $V'(x)$.

(c) Evaluate $\langle x^2 \rangle$ for the ground state of this potential $V'(x)$.

Problem 7:
(a) Show that circular orbits exist for all attractive power-law central potentials $V(r) = ar^k$.
(b) Find the radius and total energy of the circular orbit as functions of the power of *r* and the angular momentum.
(c) Find the allowable range of the exponent *k* for which the orbits are stable.

Problem 8:
A non-relativistic positron of charge e and velocity $\mathbf{v}_1$ ($v_1 << c$) impinges head-on on a fixed nucleus of charge Zq_e. The positron which is coming from far away (∞), is decelerated until it comes to rest and then accelerated again in the opposite direction until it reaches a terminal velocity $\mathbf{v}_2$. Taking radiation loss into account (but assuming it is small), find v_2 as a function of v_1 and Z.