January 2014 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

Useful integral:

$$\int ln(x)\, dx = xln(x) - x$$

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

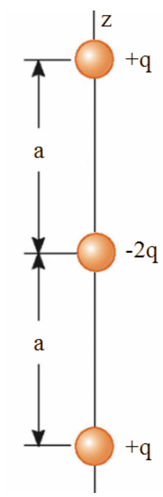
Problem 1:
An apple with a 0.2 kg mass sits on a table in equilibrium.
(a) What forces act on it? Provide their magnitude and direction.
(b) What is the reaction force to each of the forces acting on the apple?
(c) What are the action-reaction pairs?

Problem 2:
A circular lens of diameter D produces a diffraction pattern whose first minimum is at an angle $\theta_D \approx 1.2\ \lambda/D$ radians, where λ is the wavelength of the light. Two stars at a distance R away from the observer form a binary with separation R_{12}. How large must R_{12} be in order for the telescope with objective diameter D to resolve the image as two stars?

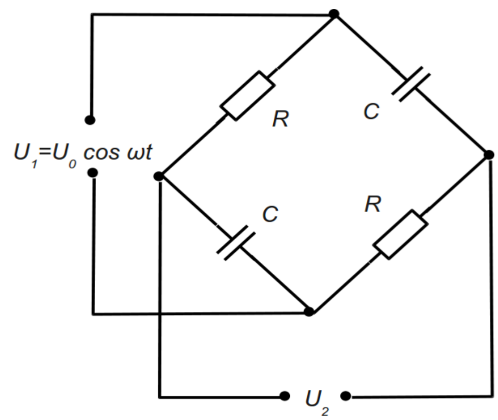
Problem 3:
In 9.0 days, the number of radioactive nuclei decreases to one-eight the number present initially.
(a) What is the **mean** life of the nuclei (in s)?
(b) If 10^9 nuclei are present, approximately haw many will decay in the next second?

Problem 4:
A bullet with mass m is fired horizontally into a wooden block with mass M which lies on a table. The bullet remains embedded in the block after the collision. The coefficient of friction between the block and table is μ, and the block slides a distance d before stopping. Find the initial speed v_0 of the bullet in terms of M, m, μ, g, and d.

Problem 5:
Consider 3 charges on the z-axis, +q at z = a, +q at z = -a, and -2q at z = 0.
(a) Write down the expression for the scalar potential $\Phi(z)$ at a position z along the z-axis for z >> a.
(b) Expand $\Phi(z)$ for a/z << 1 and show that $\Phi(z)$ can be written as $\Phi(z) \approx Q/(4\pi\varepsilon_0 z^3)$.
(c) Determine the constant Q (the quadrupole moment).



Problem 6:
For the RC circuit shown on the picture, calculate the U_2, knowing the sinusoidal change of U_1.



Problem 7:
(a) Derive an expression for the terminal speed v_t of a sphere falling in a viscous fluid in terms of the sphere's radius r and density ρ and the fluid's density ρ' and viscosity η, assuming that the flow is laminar so that Stokes' law, $F_v = 6\pi\eta rv$, is valid.
(b) Now assume that a copper sphere with a mass of 0.4 g falls with a terminal speed of 5.0 cm/s in an unknown liquid. If the density of copper is 8900kg/m^3 and that of the liquid is 2800 kg/m^3, what is the viscosity of the liquid?

Problem 8:
A proton is in the spin state $|+> = \frac{1}{4}\begin{pmatrix} 2-3i \\ i\sqrt{3} \end{pmatrix}$, where $|+> = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is the eigenstate of S_z with eigenvalue ½ħ.
(a) What is the expectation value of S_z?
(b) What is the expectation value of S_x?
(c) What is the expectation value of S_y?

Problem 9:
Calculate capacitance per unit length for a piece of coaxial cable. The coaxial cable has diameter of inner wire 1 mm and inner diameter (ID) of an outer shield 5 mm. The dielectric constant of the insulator between the two conductors is $\varepsilon = 1.5\ \varepsilon_0$.

Problem 10:
A conservative force $\boldsymbol{F}(\boldsymbol{r})$ is one for which $\oint \boldsymbol{F}(\boldsymbol{r}) \cdot d\boldsymbol{r} = 0$ for any closed path.
Use Stoke's theorem to find which of the following three forces are conservative.

(a) $F_x = 6z^3y - 20x^3y^2$, $F_y = 6xz^3 - 10x^4y$, $F_z = 18xz^2y$.
(b) $F_x = 18yz^3 - 20x^3y^2$, $F_y = 18xz^3 - 10x^4y$, $F_z = 6yz^2$.
(c) $\mathbf{F} = \mathbf{i}\, F_x(x) + \mathbf{j}\, F_y(y) + \mathbf{k}\, F_z(z)$.

Section II:
Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:
Consider a glancing (non-relativistic) elastic collision of particle A of mass m with particle B of mass 2m in which particle B is initially at rest in the lab. In the center of mass frame, particle A travels forward after the collision at a final angle of 30° with respect to its initial direction of travel.
(a) What is the final angle for particle A in the lab frame?
(b) What is the final angle of particle B in the lab frame?
Obtain numerical answers in degrees and provide a drawing.

Problem 12:
Consider the following process done on an ideal monatomic gas.
1000 J are added at constant pressure. The pressure is 1 atm $=1.01*10^5$ Pa. The system expands in the process and its temperature rises by 10 K.
(a) Find the number of moles present.
(b) Find the change in the volume ΔV.
(c) Find the entropy change in terms of the initial temperature T_0.

Problem 13:
A single-stage rocket with initial mass m_0 is launched vertically and has enough fuel, that its engine is active for t = 60 s (burnout time). The thrust ratio of the rocket defined as a ratio of the thrust of the rocket and its initial weight, $R = T/m_0g$, and is R = 2. Determine the maximum height attained by a single stage rocket for which the ratio between burnout and initial mass $m_{b0}/m_0 = 0.5$. Assume that rocket loses mass at a constant rate.

Problem 14:
A particle of mass m is in a cubical well of side L, corresponding to the potential energy function $U(r) = 0$ if $|x|$, $|y|$ and $|z| < L/2$, $U(r) = \infty$ otherwise.
(a) What is its ground-state energy?
(b) What is the energy of the first excited state? Is this energy level degenerate or non-degenerate? Explain!
(c) Suppose 20 identical, non-interacting particles of mass m and spin ½ are in this well. What is the ground state energy of this system? Is this ground state degenerate or non-degenerate? Explain!

Problem 15:
An electron with energy $E(e^-) = 8.89$ GeV collides head-on with a positron of energy $E(e^+) = 3.16$ GeV. If a single particle is created in this collision, what is its rest energy?