Spring 2012 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Mark exactly eight problems in section I and three problems in Section II.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K

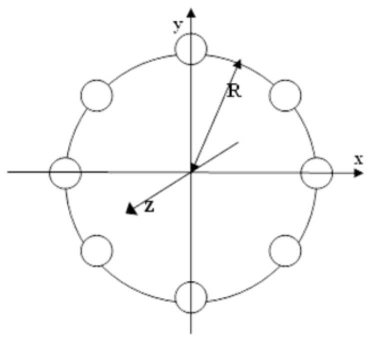
Units: 1 kcal = 4186 J

Section I:

Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

Eight small spheres with uniform surface charge (open circles) are fixed at regular intervals on a circle with radius R = 2 m in the xy-plane. Each sphere carries $Q = -5*10^{-9}$ C of negative charge. Assuming the potential of the charge configuration to be zero at infinity, what is the potential at the center of the circle?



Problem 2:

For a harmonic oscillator potential centered at x = 0 the expectation values below are given.

$$\langle x \rangle = 0 \qquad \langle p_x \rangle = 0$$

$$\langle x^2 \rangle = \frac{\hbar^2}{2m^2\omega_0^2} \qquad \langle p_x^2 \rangle = \frac{m^2\omega_0^2}{2}$$

Why is the harmonic oscillator known as a minimum uncertainty potential?

Problem 3:

A worker sitting on top of the roof of a house drops a hammer. The roof is smooth and slopes at an angle of 30^o to the horizontal. It is 10 m long and its lowest point, where it meets the vertical wall, is 10 m above the ground. How far from the house wall is the hammer when it hits the ground?

Problem 4:

What must be the initial velocity of a projectile to leave the earth? The air friction is to be neglected! (mean radius: R_{Earth} = 6371 km)

Problem 5:

What is the lowest energy a photon can have in order to produce an electron-positron pair in the presence of a nucleus of mass M?

Problem 6:
A building at a temperature T (in K) is heated by an ideal heat pump with coefficient of performance $COP_{max} = Q_{low}/(Q_{high} - Q_{low}) = T_{low}/(T_{high} - T_{low})$. The heat pump uses the atmosphere at T_0 (K) as the heat source. The pump consumes power W and the building loses heat at a rate $\alpha(T - T_0)$. What is the equilibrium temperature of the building?
Express your answer in terms of α, W and T_0.

Problem 7:
Find the electric field **E** and the electric displacement **D** produced by a thin rectangular slab with permanent uniform polarization **P** as shown in the figure. Ignore edge effects.

Problem 8:
How fast would you need to be driving such that a red light (λ_0 = 675 nm) appears yellow (λ = 575 nm)?

Problem 9:
An infinitely long charged wire with linear charge density λ is surrounded by an infinite conducting shell with inner and outer radius R_1 and R_2, respectively.
(a) Find the electric field in all regions of space.
(b) Find the surface charge density on the surfaces of the conducting shell.

Problem 10:
0.5 kg of a hot metal at 80°C is dropped into a very large pool of water at 20°C. The specific heat of the metal at constant pressure is independent of temperature, with a value of C = 100 J/(kg-K).
(a) How much does the entropy of the metal change?
(b) How much does the total entropy (of both the metal and water) change? Does it increase or decrease?

Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

(a) Define what is meant by the term "stationary state" in quantum mechanics, and explain the distinction between the time-dependent and time-independent Schrödinger equation.

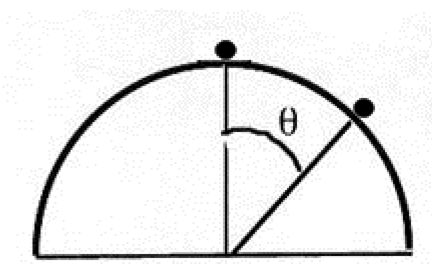
(b) At time t = 0, the wave function of a particle in one dimension is $\psi(x) = (u_1(x) + u_2(x))/\sqrt{2}$, where $u_1(x)$ and $u_2(x)$ are two solutions of the time-independent Schrödinger equation. For this particle, how does the probability density change with time?

Problem 12:

Let a particle of mass m be positioned at the "north pole" of a frictionless smooth hemisphere of radius R. After a small displacement it slides down at the hemisphere.

(a) Find the angle θ when the particle separates from the hemisphere.

(b) What is its speed in that moment?



Problem 13:

An electromagnetic wave passes through a boundary between two media with $n_1 = 1$ and $n_2 = 3$ at near-normal incidence of $\theta_i = 0.5$ degree.

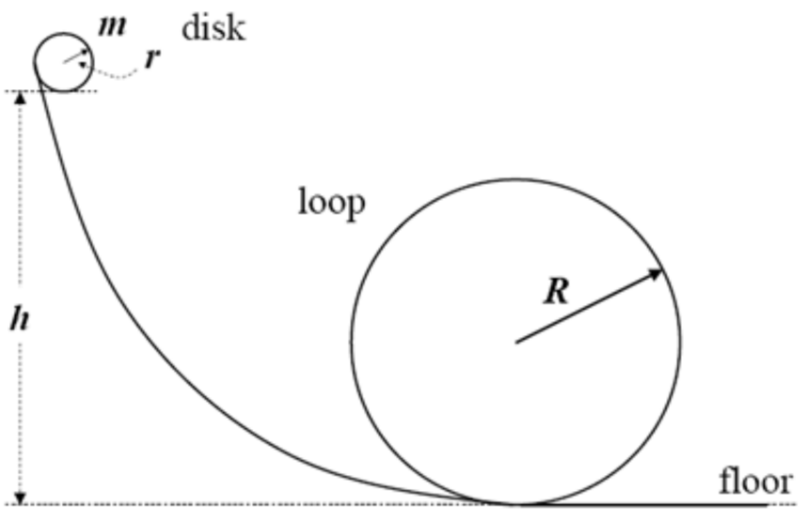
(a) Find the angle θ_t of the transmitted wave

(b) Find the reflectance and the transmittance for the wave.

Problem 14:
A disk of radius r is released from rest at a point where its center-of-mass is a distance h higher than when the disk is on the floor. The disk then rolls without slipping down an inclined track. At the floor level it rolls into a circular loop of radius R.

Find the minimum starting height h needed to ensure that the disk will remain in contact with the track at all times for the case that r = 1/3 R. Express the answer in terms of R.

The moment of inertia for the disk is $I = mr^2/2$.



Problem 15:
A conducting circular loop made of wire of diameter d, resistivity ρ , and mass density ρ_m is falling from a great height h in a magnetic field with a component $B_z = B_0(1 + kz)$, where k is some constant. The loop of diameter D is always parallel to the x-y plane. Disregard air resistance, and find the terminal velocity of the loop.