January 2018 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

Useful integral:
$\int \sin^2(x)dx = x/2 - \sin(2x)/4$

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

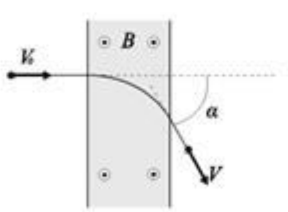
Problem 1:
Consider a particle of mass m in a one-dimensional potential energy well
$U(x) = \frac{1}{2} kx^2$ for $x > 0$ and $U(x) = \infty$ for $x < 0$. What is the ground-state energy?

Problem 2:
A 6.00 μF capacitor that is initially uncharged is connected in series with a 5.00 Ω resistor and an emf source with ε = 50.0 V and negligible internal resistance. At the instant when the resistor is dissipating electrical energy at a rate of 300 W, how much energy is been stored in the capacitor?

Problem 3:
Show that the second Kepler law, the radius vector of a planet covers equal areas in equal times, follows directly from the angular momentum conservation law.

Problem 4:
A laser beam is incident at an angle of 30.0° to the vertical onto a solution of corn syrup in water. If the beam is refracted to 19.24° to the vertical,
(a) What is the index of refraction of the syrup solution?
Suppose that the light is red, with vacuum wavelength 632.8 nm. Find its
(b) wavelength,
(c) frequency, and
(d) speed in the solution.
Give numerical answers.

Problem 5:
A particle with the mass $m = 6.68 \cdot 10^{-27}$ kg and electrical charge $q = 3.2 \cdot 10^{-19}$ C is entering a region with a uniform magnetic field as shown in the figure. The field strength is 0.03 T. The particle is deflected by an angle α = 0.8 rad. How much time does it take for the particle to cross the magnetic field region? Give a numerical answer?



Problem 6:

A particle of mass m is in the ground state of an infinite square well ($U = 0$ for $0 < x < a$ and $U = \infty$ otherwise).

At time $t = 0$, the right "wall" (i.e. at $x = a$) shifts to $x = 2a$. A measurement of the energy of the particle is made just after the wall shifts. (Assume that all this happens so quickly so the spatial wave function of the particle does not change). What is the probability that the energy measurement yield a value EXACTLY the same as the energy of the ground state of the original well?

Problem 7:

Two balls of equal masses m and equal speeds $v = 0.8\,c$ approach one another head-on. Their collision is perfectly inelastic, so they stick together and form a single body of mass M.

(a) What is the mass M?

(b) What is the velocity of the final body?

Problem 8:

A long, straight, solid cylinder, oriented with its axis in the z-direction, carries a current whose current density is **j**. The current density, although symmetrical about the cylinder axis, is not constant and varies according to the relationship

$\mathbf{j} = (b/r)\, e^{(r-a)/\delta}\, \mathbf{k}$ for $r < a$, $\mathbf{j} = 0$ for $r > a$,

where the radius of the cylinder is $a = 5.00$ cm, r is the radial distance from the cylinder axis, b is a constant equal to 600 A/m, and δ is a constant equal to 2.50 cm.

(a) What is the total current I_0 passing through the entire cross section of the wire?

(b) Using Ampere's law, derive an expression for the magnetic field **B** in the region $r \geq a$. Express your answer in terms of I_0

(c) Obtain an expression for the current I contained in a circular cross section of radius $r \leq a$ and centered at the cylinder axis. Express your answer in terms of I_0, rather than b.

(d) Using Ampere's law, derive an expression for the magnetic field **B** in the region $r \leq a$. Express your answer in terms of I_0, rather than b.

(e) Evaluate the magnitude of the magnetic field at $r = \delta$, a, and 2a.

Problem 9:

^{238}Pu decays by α-emission with a half-life of 87 years.
$^{238}Pu \rightarrow {}^{234}U + \alpha$.
The half-life of ^{234}U is much longer, $3.5*10^5$ years (ignore this decay).
The heat produced in this decay can be converted into useful electricity by radio-thermal generators (RTG's). The Voyager 2 space probe, which was launched in August 1977, flew past four planets, including Saturn, which it reached in August 1981.
How much plutonium would an RTG on Voyager 2 with 5.5 % efficiency have to carry at the start to deliver at least 395 W of electric power when the probe flies past Saturn?

Masses:
^{4}He: 4.002603 u,
^{234}U: 234.040947 u
^{238}Pu: 238.049555 u
Conversion: 1 u = 931.5 MeV

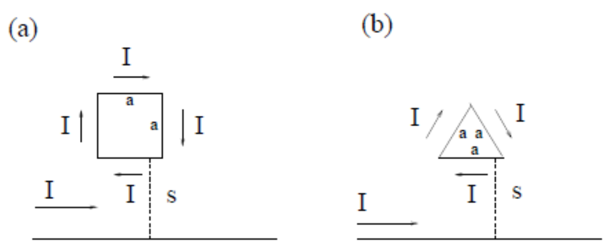
Problem 10:

For a proton of mass $m = 1.66*10^{-27}$ kg, find the ratio p/v, if the proton is moving with speed
(a) $3*10^7$ m/s and
(b) $2.7*10^8$ m/s.
(c) Compare the kinetic energies of the proton in both cases according to a non-relativistic and a relativistic calculation.

Section II:
Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

(a) Consider an infinite straight wire placed along the x-axis and carrying a current **I** = I **i**, i.e. the current flows to the right. Provide the magnetic field **B** produced by the wire at point P = (0, y, 0).
(b) Now find the force on a square loop placed as shown in panel (a) of the figure near the infinite straight wire. Both the loop and the wire carry a current of magnitude I. Provide the magnitude and the direction of the force.
(c) Now find the force on a triangular loop placed as shown in panel (b) of the figure near the infinite straight wire. Both the loop and the wire carry a current of magnitude I. Provide the magnitude and the direction of the force.



Problem 12:

A particle in a spherical potential is known to be in an eigenstate of L^2 and L_z with eigenvalues $l(l + 1)\hbar^2$ and $m\hbar$, respectively. Prove that the expectation values between $|l\ m\rangle$ states satisfy
$\langle L_x\rangle = \langle L_y\rangle = 0$, $\langle L_x^2\rangle = \langle L_y^2\rangle = (l(l + 1)\hbar^2 - m^2\hbar^2)/2$.

Problem 13:

A simple pendulum of mass m and length l is free to swing in the xy-plane. It is fixed to a support of mass M that can move freely in the horizontal direction. Using the coordinates x, the horizontal position of the support, and θ, the angle of the pendulum from vertical, write down the Lagrangian for this system.

Problem 14:

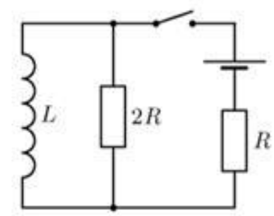
Consider a two-level system $|\Phi_a\rangle$, $|\Phi_b\rangle$ with $\langle\Phi_i|\Phi_j\rangle = \delta_{ij}$. Show that an "entangled", two-particle state of the form

$\alpha|\Phi_a(1)\rangle|\Phi_b(2)\rangle + \beta|\Phi_b(1)\rangle|\Phi_a(2)\rangle$, $\alpha, \beta \neq 0$,

CANNOT be written as a product state $|\psi_r(1)\rangle|\psi_s(2)\rangle$ for any one particle states $|\psi_r\rangle$ and $|\psi_s\rangle$.

Problem 15:

An electric circuit with only ideal elements is shown in the figure. No current flows before, at some moment, the switch is closed for a short time. Right after the switch is closed, the current through resistor 2R is I_0. When the switch is opened again the current through the same resistor is $2I_0$.



(a) How long was the time interval for which the switch was closed?

(b) How much energy was dissipated by the system after the switch was opened again?