January 2017 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

Units: 1 Gauss = 10^{-4} T

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:
A charge q moves with velocity **v** in a magnetic field **B**.
(a) What is the magnetic force exerted on q?
(b) If **B** is constant, and the only force acting on the charge is the magnetic force, describe possible trajectories.
(c) Explain why the magnetic force does no work.

Problem 2:
Is the ground state of the infinite square well an eigenfunction of the momentum operator?
If so, what is the eigenvalue of the momentum of the particle in that state?
If not, why not?

Problem 3:
Assume the sun has a mass of $2*10^{30}$ kg, and the earth has a mass of $6*10^{24}$ kg.
Assume that both contain an equal number of protons and neutrons, and each nucleon has a mass of $2*10^{-27}$ kg.
Assume the Coulomb force constant is $9*10^9$ Nm^2/C^2, the electron charge is $2*10^{-19}$ C, and the gravitational force constant is $7*10^{-11}$ Nm^2/kg^2.

(a) Assume an equal number of electrons is added to the sun and the earth to cancel the gravitational force between the two and produce a net force of zero. How many electrons are added to each body? Do you need to consider the mass of the electron? Why do you not need a numerical value for the astronomical unit?
(b) What is the fractional increase in the number of electrons required per nucleon on the earth?

Problem 4:
A particle of mass m moves in a horizontal circle of radius r on a rough table. It is attached to a string fixed at the center of the circle. The speed of the particle is initially v_0.
(a) After completing one full trip around the circle, the particle's speed is now $\frac{1}{2}v_0$. What is the work done by friction during this one revolution?
(b) What is the coefficient of kinetic friction?
(c) How many more revolutions will the particle make before it comes to rest?

Problem 5:
Consider a washer (a flat circular disk with a circular hole in the center) with height h. The radius of the circular hole is *a* and the radius of the washer is 2*a*. The washer is made of material of resistivity ρ. A small slit is cut out of the washer and wires are connected to both ends. What is the resistance of the washer in this configuration?

Problem 6:
(a) A spin ½ particle can have $S_z = \pm\frac{1}{2}\hbar$. Construct states for two such particles that are symmetric or anti-symmetric under interchange and find their total spin **S**.
(b) Assuming that ^3H and ^3He consist of protons and neutrons in L = 0 states, find their total angular momentum.

Problem 7:
Assume that a star with a radius of 10^7 km has a purely dipolar magnetic field and that the magnetic field strength in its interior is 100 Gauss. The star collapses to a neutron star with a radius of 10 km. Assume that even though mass is lost during this transformation, magnetic flux through an area bounded by the equator is conserved.
(a) What is the field strength in the interior of the neutron star after the collapse?
(b) What is the magnetic energy density in the interior of the neutron star?
How does this compare with the energy density from the gravitational field, 2 GeV/fm^3 and what does that mean in terms of the ability of the magnetic field to modify the neutron star's structure?

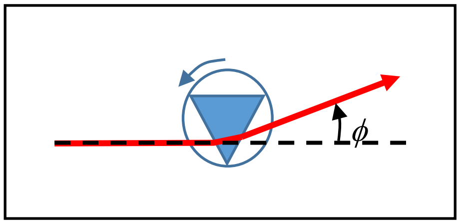
Problem 8:
A particle of mass m moves under the influence of a potential energy function that is given by
$U(x) = (1 - ax)\exp(-ax)$
with $a > 0$.
(a) Determine the location of the equilibrium point(s).
(b) Determine the nature of the equilibrium (stable or unstable).

Problem 9:
A 40 kg girl gets on her 10 kg wagon on level ground. She has two 5 kg bricks with her. She throws the bricks horizontally off the back of the wagon one at a time with a speed of 7 m/s relative to herself.
(a) How fast is she moving after throwing the second brick?
(b) How fast would she be going if she had thrown both bricks at the same time at a speed of 7 m/s relative to herself?

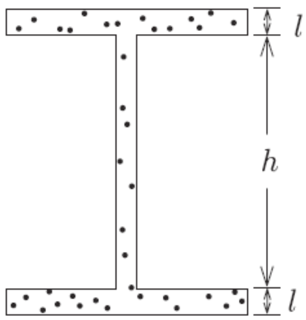
Problem 10:
The average density of the Moon is 0.604 times that of the Earth and the mass of the Moon is 0.012 times that of the Earth. If a baseball, with an approximate mass of 150 grams, is thrown straight upward with an initial velocity of 20 m/s from the surface, how high will it go on the Earth and on the Moon, neglecting any effect of friction?

Section II:
Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:
For a symmetrical prism (one in which the apex angle lies at the top of an isosceles triangle), the total deviation angle ϕ of a light ray is minimized when the ray inside the prism travels parallel to the prism's base.
Assume that a beam of light passes through a glass equilateral prism with refractive index 1.5. The prism is in air and is mounted on a rotation stage, as shown in the figure. When the prism is rotated, the angle by which the beam is deviated changes. What is the minimum angle ϕ by which the beam is deflected?



Problem 12:
N atoms, each of mass m, of an ideal monatomic gas occupy a volume consisting of two identical chambers connected by a narrow tube, as illustrated on the right. There is a gravitational field with acceleration g directed downward in the figure. The gas is in thermal equilibrium at temperature T. The height h of the upper chamber above the lower chamber is much greater than the height l of either chamber. The volume of the tube is negligible compared with that of the chambers. Assume that $mgl \ll k_BT$, but not that there is any particular relation between mgh and k_BT. Treat the system classically and the atoms as having no internal degrees of freedom.



(a) Calculate the number of atoms in the upper chamber in terms of N, m, g, h, and T.
(b) Calculate the total energy (kinetic and potential) of the system in terms of the same parameters.
(c) From the total energy obtain an expression for the specific heat of the system as a function of the temperature.

Problem 13:

An alpha particle of mass m = 6.64×10^{-27} kg and charge $q = 2|q_e| = +3.20 \times 10^{-19}$ C is accelerated from rest in a vacuum by a uniform electric field of 50 million volts over a distance of 60 m.

(a) Determine v/c, its speed as a fraction of the speed of light, at the end of its acceleration.
(b) Determine its de Broglie wavelength at the end of its acceleration.

Problem 14:

An electron in the hydrogen atom occupies the combined position and spin state

$$R_{21}(r)\left(\sqrt{\frac{1}{3}}Y_1^0(\theta,\varphi)\chi_+ + \sqrt{\frac{2}{3}}Y_1^1(\theta,\varphi)\chi_-\right).$$

(a) If you measure L^2, what value(s) might you get, and with what probability(ies)?
(b) If you measure L_z, what value(s) might you get, and with what probability(ies)?
(c) If you measure S^2, what value(s) might you get, and with what probability(ies)?
(d) If you measure S_z, what value(s) might you get, and with what probability(ies)?
(e) If you measured the position of the electron, what is the probability density for finding the electron at r, θ, φ in terms of the variables given above.
(f) If you measured both S_z and the distance of the electron from the proton, what is the probability per unit length for finding the particle with spin up a distance r from the proton in terms of the variables given above?

Useful integral:

$$\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta \, |Y_l^m(\theta,\varphi)|^2 = 1$$

Problem 15:

Consider a cyclic (reversible) heat engine. The T-S diagram for its operation is shown below.

(a) What is the efficiency of this engine?
(b) What does the corresponding diagram for a Carnot engine look like in the T-S plane?