August 2015 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:
A metal wire of length L is formed into a loop, either square or circular. A current I flows through the loop and it is placed in a uniform magnetic field. Which loop shape results in a larger torque? Why?

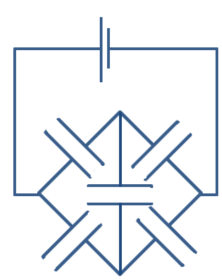
Problem 2:
A prankster drops a water-filled balloon out of a window. The balloon is released from rest at a height of 10 m above the ears of an innocent man who is the target. Then, because of a guilty conscience, the prankster shouts a warning after the balloon is released. The warning will do no good, however, if shouted after the balloon reaches a certain point, even if the man could react infinitely quickly. Ignoring the effect of air resistance on the balloon, determine how far above the innocent man's ears this point is. The speed of sound is 340 m/s.

Problem 3:
Consider a system that is made of an isolated spherical conductor of radius R and an immobile negative point charge –q located at the center of the conductor. An additional total charge +Q is placed on the conductor. Find the potential at all points inside and outside of the conductor once electrostatic equilibrium has been reached. Set the potential at infinity to zero.

Problem 4:
For $l = 2$ in the hydrogen atom:
(a) What is the minimum value of $L_x^2 + L_y^2$?
(b) What is the maximum value of $L_x^2 + L_y^2$?
(c) What is $L_x^2 + L_y^2$ for $l = 2$ and $m_l = 1$? Can either L_x or L_y be determined from this?
(d) What is the minimum value of n this state can have?

Problem 5:

Consider the following circuit consisting of identical capacitors with capacitance C. What is the effective capacitance of the circuit?



Problem 6:

A certain species of ionized atoms produces an emission line spectrum according to the Bohr model, but the number of protons in the nucleus is unknown. A group of lines in the spectrum forms a series in which the shortest wavelength is 22.79 nm and the longest wavelength is 41.02 nm. Find the next-to-longest wavelength in the series of lines.

Problem 7:

In a solar collector, water flows through pipes that collect heat from an area of 10 m^2. The collector faces the Sun and the intensity of sunlight incident on it is 2000 W/m^2. At what rate (in kg/minute) should the water circulate through the pipes so that it is heated by 40^{o}C, if the collector efficiency is 30%? (The specific heat of water is c = 4200 J/(kg ^{o}C)).

Problem 8:

Find the rotational energy levels of a diatomic molecule with atoms of mass m_1 and m_2. Take r_1 and r_2 to be the distances from atoms 1 and 2 to the center of mass.

Problem 9:

In a volume of space V the electric field is $\vec{E} = c(2x^2 - 2xy - 2y^2)\hat{x} + c(y^2 - 4xy - x^2)\hat{y}$, where c is a constant.

(a) Verify that this field represents an electrostatic field.

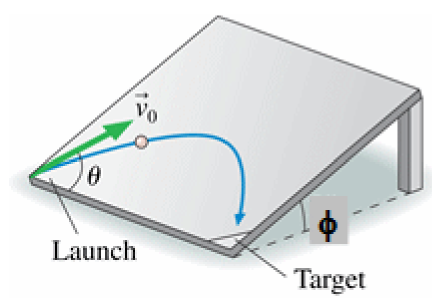
(b) Determine the charge density ρ in the volume V consistent with this field.

Problem 10:

In a carnival game, you have to throw a ball with speed v_0 at an angle θ in order to hit a target on the other side of the platform, located a distance h away.

The platform is inclined at an angle ϕ.

Find the angle θ in terms of the other variables.



Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

A supernovae at a distance d from Earth explodes, and photons and neutrinos are emitted. What is the difference between the arrival time on Earth for the photons and neutrinos if neutrinos have an energy E_ν? Assume that the neutrino mass m fulfills $mc^2/E_\nu << 1$.

Give a numerical answer for $d = 10^5$ light years and $mc^2/E_\nu = 10^{-6}$.

Problem 12:

A damped oscillator satisfies the equation $d^2x/dt^2 + 2\gamma\, dx/dt + \omega_0^2 x = 0$, where γ and ω_0 are positive constants with $\gamma < \omega_0$ (under-damping). Assume that the equation of motion of a particle of mass m is given by this equation. At time t = 0, the particle is released from rest at the point x = a.

(a) Find x as a function of time t and sketch x as a function of t.

(b) Find all the turning points of the motion and determine the ratio of successive maximum values of x(t).

(c) Re-do part(a) for the case of critical damping, i.e. when $\gamma = \omega_0$.

Problem 13:

At time, $t_0 = 0$ there are N_A atoms of a radioactive nucleus A in the decay sequence A -> B -> C.

(a) Show that the number of atoms of the radioactive nucleus B at time t is

$N_B(t) = \frac{N_{A0}\lambda_A}{\lambda_B - \lambda_A}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]$, where λ_A and λ_B are the decay constants for nuclei A and B.

(b) Find approximate expressions for the decay rate if

(i) $\lambda_A >> \lambda_B$, (ii) $\lambda_A << \lambda_B$, (iii) $\lambda_A \approx \lambda_B$.

Problem 14:

Find the threshold energy for the process $\gamma + p \rightarrow \pi + p$ in which a single π–meson, or pion, is produced when an energetic photon (or gamma ray) strikes a proton at rest. The threshold energy is the minimum energy of the photon. The rest energy of a π^0 is 135 MeV, and that of the proton is 938 MeV.

Problem 15:

Consider a particle in a simple two level system with eigenstates ψ_1 and ψ_2 such that $H\psi_1 = E_1\psi_1$ and $H\psi_2 = E_2\psi_2$. Derive an expression for the specific heat per particle c_V of the system as a function of temperature.