August 2013 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\,400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\,7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\lambda)) - 1]^{-1}$

Useful integral:

$$\int \frac{dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{\pm x}{a^2\sqrt{(x^2 \pm a^2)}}$$

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:
A uniform rod of length b and mass m stands vertically upright on a rough floor and tips over. What is the rod's angular speed just before it hits the floor?

Problem 2:
A disk with radius r = 0.10 m is oriented with its normal unit vector **n** at an angle of 30^o to a uniform electric field **E** with magnitude 2.0×10^3 N/C.
(a) What is the electric flux through the disk?
(b) What is the flux through the disk if it is turned so that its normal is perpendicular to **E**?
(c) What is the flux through the disk if its normal is parallel to **E**?

Problem 3:
Calculate the entropy change of an ideal gas that undergoes a reversible isothermal expansion from volume V_1 to V_2.

Problem 4:
Consider an automobile traveling north at 80 km/hr. It slows down, makes a right turn, and speeds up to 80 km/hr. If it takes 30 seconds for the car to slow, turn, and speed up, what is the average acceleration of the automobile over these 30 seconds? Give both magnitude and direction of the average acceleration. Give the magnitude in units of m/s^2.

Problem 5:
Find eigenvalues and normalized eigenvectors of the Hamiltonian

$$\hat{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Problem 6:
A uniformly solid sphere of mass M and radius R is fixed at distance h above a thin infinite sheet of mass density ρ_s (mass/area). With what force does the sphere attract the sheet?

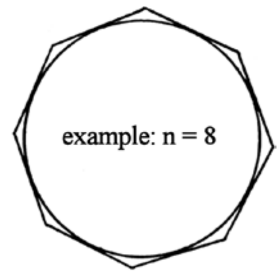
Problem 7:
As a rocket ship passes Earth at speed (3/5)c, clocks on Earth and on the ship are synchronized at t = 0. The rocket ship sends a light signal back to Earth when its clock reads one hour.
(a) According to Earth's clock, when was the signal sent?
(b) According to Earth's clock, how long after the rocket passed did the signal arrive back on Earth?
(c) According to the ship clock, when did the signal arrive back on Earth?

Problem 8:

A circuit in the form of a regular polygon of n sides is circumscribed about a circle of radius a.

(a) If it is carrying a current I, find the magnitude of the magnetic field B at the center of the circle in terms of μ_0, n, I, and a.

(b) Find B at the center of the circle as n is indefinitely increased.



Problem 9:

The wavelengths of the visible spectrum are approximately 400 nm (violet) to 700 nm (red).

(a) Find the angular width of the first-order visible spectrum produced by a plane diffraction grating with 600 slits per mm when white light falls normally on the grating.

(b) Show that the violet end of the third-order spectrum overlaps the red end of the second-order spectrum.

Problem 10:

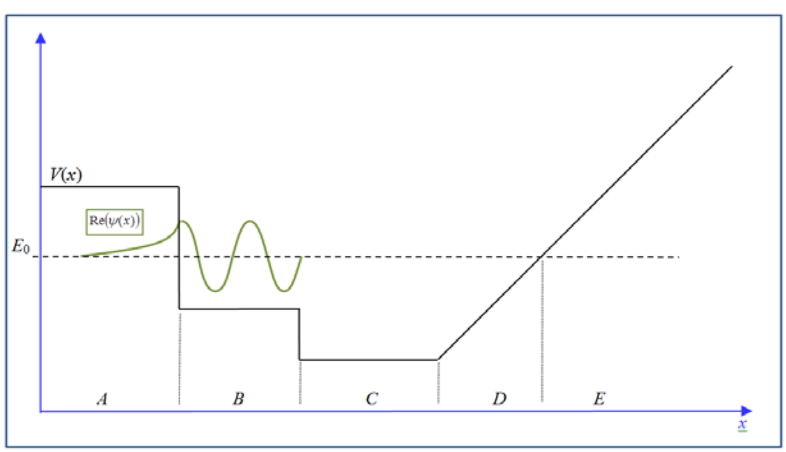
Consider a quantum mechanical particle of energy E_0 in one-dimension, with potential energy function V(x) as shown in the figure. In principle, the time-independent Schrödinger equation could be solved numerically to exactly find the wave function, but an understanding of its physical interpretation can be used to determine its amplitude and phase behavior.

(a) Complete the sketch of the real part of a possible wave function Re(ψ(x)) for zones C, D, and E that is consistent with the given potential.

(In doing so, pay attention to the following two questions.)

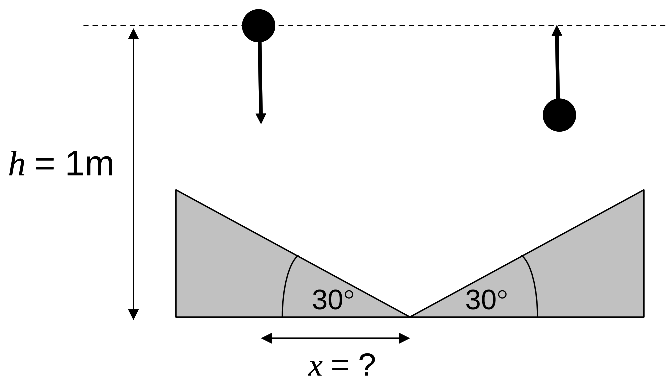
(b) Is the amplitude in zone C bigger or smaller than that in zone B? Why?

(c) Is the frequency of oscillations in zone C higher or lower than that in zone B? Why?

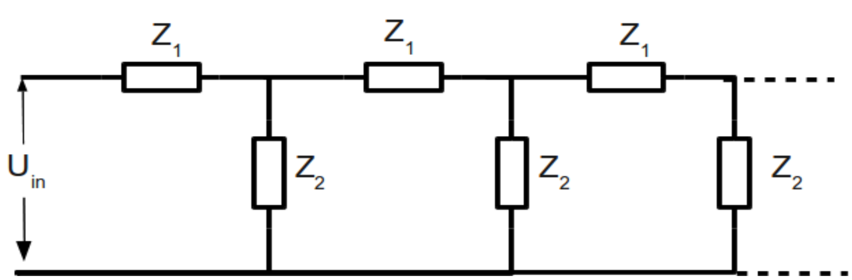


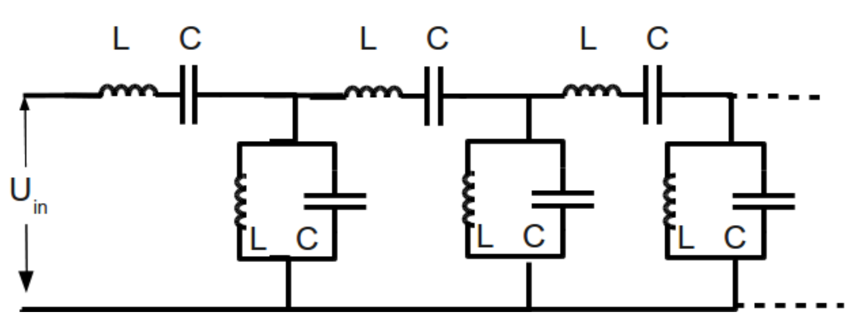
Section II:
Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:
Two wedges are placed mirror symmetrically so that the tip of one touches the tip of another as shown in the figure below. The surface of each wedge is at angle $\theta = 30$ degrees relative to the ground. A small elastic ball is dropped from height h = 1 m with zero initial velocity. How far from the tips of the two wedges (x) must the small ball be dropped, so that after bouncing from the two wedges will reach the same height from where it was dropped? Neglect any friction from air and consider the bouncing of the ball to be completely elastic.



Problem 12:
(a) Calculate the impedance Z of an infinite chain of elements with impedances Z_1 and Z_2, as shown in the top figure.
(b) Calculate Z_1 and Z_2 for the specific case shown in the bottom figure.





Problem 13:
An alpha particle of mass m = 6.64×10^{-27} kg and charge q = –2e = $+3.20\times10^{-19}$ C is accelerated from rest in a vacuum through a potential difference of 50 million volts over a distance of 60 m.
(a) Determine v/c, its speed as a fraction of the speed of light at the end of its acceleration.
(b) Determine its de Broglie wavelength at the end of its acceleration.

Problem 14:
(a) Find an expression for the variation in atmospheric pressure P with elevation h above sea level assuming that the temperature of the atmospheric air T and the acceleration due to gravity g are both constant with elevation and that the atmospheric air is an ideal gas with molar mass M.

(b) Now if the pressure at sea level is P_0 = 1 atm, calculate the atmospheric pressure at the summit of Mount Everest at an altitude of 8863 m above sea level, assuming that the temperature remains constant at T = 0^{o}C, and the molar mass of air is M = 28.8×10^{-3} kg/mol. Give your result in atm.

Problem 15:
For a satellite orbiting a planet, transfer between coplanar circular orbits can be affected by an elliptic orbit with perigee and apogee distances equal to the radii of the respective circles as shown in the Figure below. This ellipse is known as Hohmann transfer orbit.
Assume a satellite is orbiting in a circular orbit of radius r_p with circular orbit speed v_c. It is to be transferred into a circular orbit with radius r_a.
(a) Find E_H/E_c, the ratio of the total energies of the satellite in the Hohmann and the initial circular orbit.
(b) Determine the equation for the ratio of the speeds v/v_c as a function of the ratio of the distances r/r_p from the focus for the Hohman transfer orbit. Evaluate v/v_c at $r = r_p$.