Fall 2012 Qualifying Exam

Part II

Mathematical tables are allowed. Formula sheets are provided.

Calculators are allowed.

Please clearly mark the problems you have solved and want to be graded.
Do only mark the required number of problems.

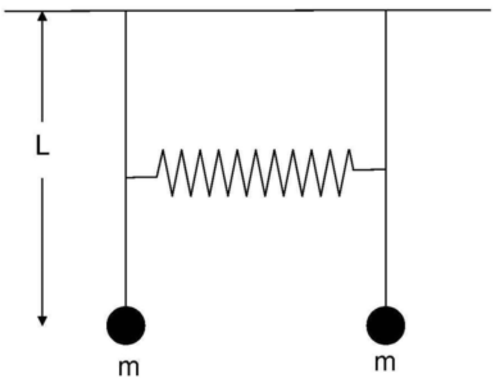
Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K

Solve 6 out of the 8 problems! (All problems carry the same weight)

Problem 1:

Two pendula, each of which consists of a weightless rigid rod length of L and a mass m, are connected at their midpoints by a spring with spring constant k. Consider only small displacements from equilibrium.



(a) What are the frequencies of the normal modes of this system? Briefly describe these modes.
(b) At $t = 0$ the right pendulum is displaced by an angle θ to the right while the left pendulum remains vertical. Both pendula are released from rest. Describe the subsequent motion.

Problem 2:

A rotating neutron star slows down due to dipole radiation. Assume that a neutron star has a surface magnetic field strength of 100 Mega Tesla at the poles, a radius of 10 km, and nuclear density. The neutron star's magnetic axis is tilted by 45 degrees relative to the axis of rotation of the neutron star. How long will it take for the neutron star's rotation rate to decrease by a factor of 3 if the initial rotation rate is 10^5 revolutions per second?

Problem 3:

Some organic molecules have a triplet ($S = 1$) excited state that is located at an energy Δ above the singlet ($S = 0$) ground state. Consider an ensemble of N such molecules where N is of the order of Avogadro's number
(a) Find the average magnetic moment $<\mu>$ per molecule in the presence of a magnetic field B at temperature T. Assume Boltzmann statistics. You may also assume that Δ is large compared to the field-induced level splitting.
(b) Show that the magnetic susceptibility $\chi = N\, d<\mu>/dB$ is approximately independent of Δ when $k_B T >> \Delta$.

Problem 4:

Suppose we have two circuits with a current I_1(t) in circuit 1 and I_2(t) in circuit 2. Show that the work done in establishing the two currents in the two circuits is

$$W = \frac{L_1}{2} I_1^2 + \frac{L_2}{2} I_2^2 + M I_1 I_2,$$

where L_1 and L_2 are the self-inductances of the respective circuits and M is the mutual inductance. Ignore the work done in heating up the wires.

Problem 5:

(a) An electron is located in the ground state in a 1D potential well,

$$U(x) = 0,\ 0 < x < L,\ U(x) = \infty \text{ elsewhere.}$$

Instantaneously the well becomes 1.5 times wider. What is the probability for the electron to go directly to ground state in this new well?

(b) An electron is located in the ground state in a 1D potential well,

$$U(x) = 0,\ 0 < x < L,\ U(x) = \infty \text{ elsewhere.}$$

For a time interval Δt, the well bottom is disturbed, and the potential energy function becomes $U(x) = 0$, $0 < x < L/2$, $U(x) = U_0$, $L/2 < x < L$, with $U_0 <<$ than the ground state energy. After the time interval Δt the disturbance is removed. What is the probability for the electron to be in the first excited state of the well at a time $t = 2\Delta t$?

Problem 6:

In one dimension, a particle is acted upon by an attractive force

$$F = -kx^3.$$

(a) Show that the period for the motion of this particle is inversely proportional to the amplitude.

(b) By contrast show that the period for the motion of a particle subjected to the force $F = -kx$ is independent of the amplitude.

Problem 7:

The spin Hamiltonian for an electron (a spin-1/2 particle), in an external magnetic field is given by

$$\hat{H} = -\gamma \hat{\vec{S}} \cdot \vec{B} \ ,$$

where the gyromagnetic ratio $\gamma = -e/mc$, e being the magnitude of the charge of the electron.

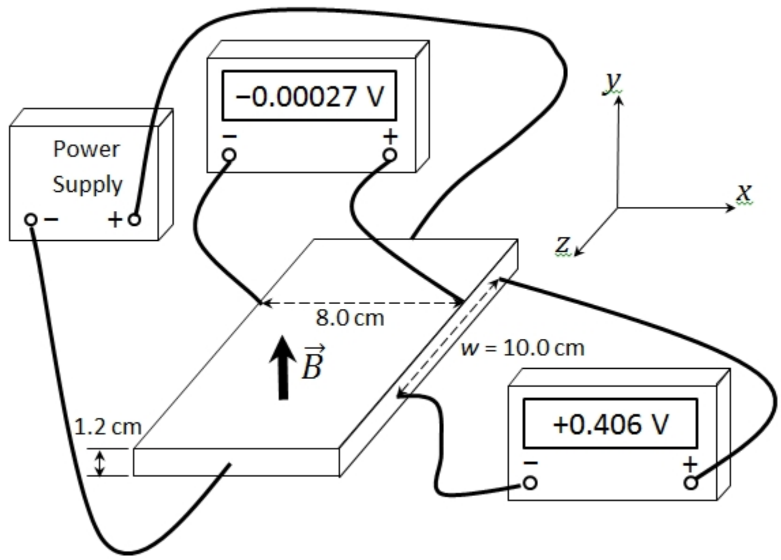
Suppose that the magnetic field consists of two components along the z and y axes, respectively,

$$\text{i.e., } \vec{B} = B_0\, \vec{e}_z + B_2 \vec{e}_y \ .$$

Let us consider the case that $B_2 << B_0$. Using perturbation theory, evaluate the possible energies of the electron to second order in the ratio B_2/B_0.

Problem 8:

A slab of unknown material is connected to a power supply as shown in the figure below. There is a uniform magnetic field of 0.8 Tesla pointing in the +y direction and perpendicular to the upper and lower faces of the slab throughout this region. Two voltmeters are connected to the slab and read steady voltages as shown on the displays in the figure. (Recall that a voltmeter produces a positive numeric reading if its positive-labeled connection is at a higher potential than its negative-labeled connection.) The connections across the slab are carefully placed directly across from each other. The distance $w = 0.10$ m. Assume that there is only one kind of mobile charges in this material, but we don't know whether they are positive or negative.



(a) Determine and state the (previously unknown) sign of the mobile charges, and state which way these charges move inside the slab (referred to the coordinate system defined in the diagram).

(b) In the steady state, the current moves straight along the slab, so the net sideways (transverse) force on a moving charge must be zero. Use this fact to determine the drift speed $\bar{v}$ of the mobile charges.

(c) Knowing the drift speed, determine the mobility u of the mobile charges. The mobility is defined as the ratio between the drift speed and the internal electric field that drives the current.

(d) The current running through the slab was measured to be 0.3 ampere. If each mobile charge is singly charged ($|q| = e$), what is the density of mobile charges in this material?

(e) What is the resistance in ohms of a 1 m length (measured along the *z* direction) of this material, having the same *x-y* cross-section?