Fall 2011 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Mark exactly eight problems in section I and three problems in Section II.

Physical Constants:

Planck constant: $h = 6.62606896 \times 10^{-34}$ Js, $\hbar = 1.054571628 \times 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 \times 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 \times 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 \times 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 \times 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 \times 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 \times 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 \times 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 \times 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 \times 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 \times 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 \times 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 \times 10^{-3}$ m K

Units: 1 kcal = 4186 J

Section I:

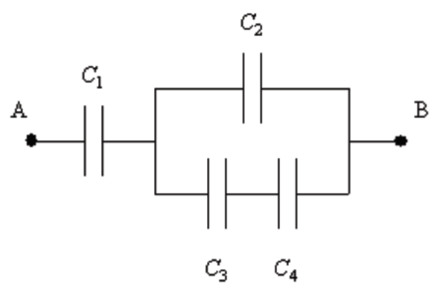
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

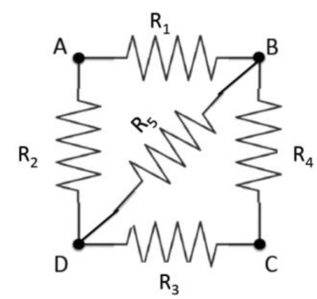
Problem 1:

A proton collides with a neutron (mass very similar to the mass of the proton) to form a deuteron. What will be the velocity of the deuteron if it is formed from a proton moving with a velocity of $7.0*10^6$ m/s to the left and a neutron moving with a velocity of $4.0*10^6$ m/s to the right?

Problem 2:

(a) Four capacitors are connected as shown in the figure.
$C_1 = C_2 = C_3 = C_4 = 1$ μF.
What is the total capacitance between points A and B?
(b) Five identical 1 Ω resistors are joined and form the four sides of square and its diagonal. What is the resistance between points A and B?





Problem 3:

Cerenkov radiation is given off when a particle moves in a medium at a speed greater than the speed of light in that medium. What is the minimum kinetic energy (in eV) that an electron (mc^2 = 511 keV) must have while traveling in crown glass (n = 1.52) in order to create Cerenkov radiation?

Problem 4:

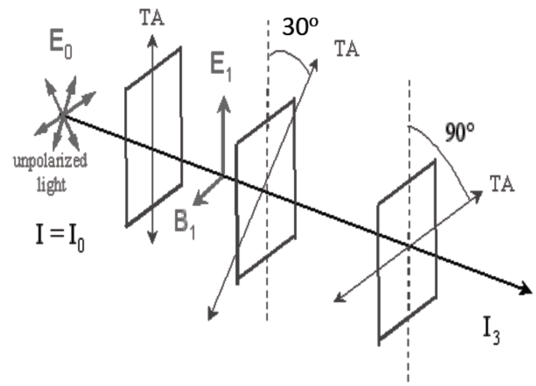
To measure magnetic fields, rotating coils are often used. A circular coil of radius 1 cm and with 100 turns is rotated at 60 Hz in a magnetic field. The induced emf in the coil has a maximum value of 12.3 V. Calculate the intensity of the field.

Problem 5:

Two spaceships, A and B, are moving along a line in opposite directions. An observer on Earth measures the speed of spaceship A to be $2*10^8$ m/s and the speed of B to be $1*10^8$ m/s.
When the captain of spaceship A receives a collision warning, the two ships are separated by $3*10^9$ m according to spaceship A's measurement How long does spaceship A's captain have to avoid a collision?

Problem 6:

A unpolarized electromagnetic wave is incident on a series of three linear polarizers, the second with the polarization angle rotated at 30° and the third at 90° with respect to the first polarizer. If the initial intensity of the unpolarized light is I_0, what is the intensity I_3 transmitted by the stack?



Problem 7:

Consider the vector field

$$\vec{E}(x, y, z) = a(x^2 - y^2 + z^2, z^2 - 2xy, 2zy + 2zx)$$

where *a* is a constant expressed in the appropriate units.

(a) Is this field irrotational?

(b) What is the corresponding charge density?

Problem 8:

An electric coffee pot contains 2 liters of water which it heats from 20° C to boiling in 5 minutes. The supply voltage is 120V and each kWh costs 10 cents. Calculate

(a) the electric power converted,

(b) the cost of making ten pots of coffee,

(c) the resistance of the heating element, and

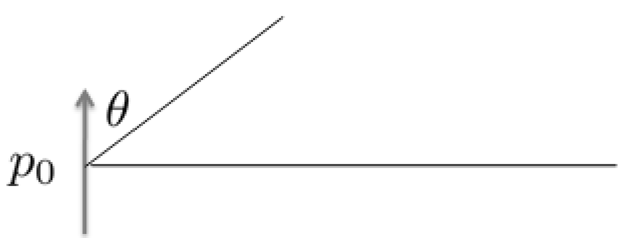
(d) the current in the element.

Problem 9:

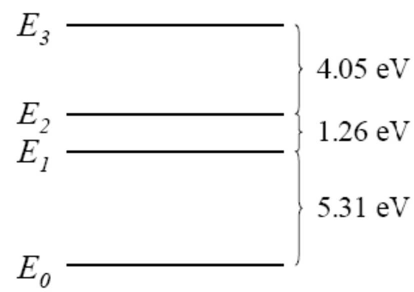
An electric dipole p_0 vibrates with frequency ω.

(a) How will the total radiated power change if the frequency is doubled?

(b) Find the ratio of the differential power in the direction of θ = 45° to the dipole's axis to the differential power in the direction perpendicular to the axis.



Problem 10:
The energy differences between the lowest four energy levels of a two-dimensional system are displayed below.



(a) Show that the energy differences are consistent with the system being a two-dimensional harmonic oscillator.
(b) What is the energy difference between the level with energy E_3 and the next higher energy level E_4?

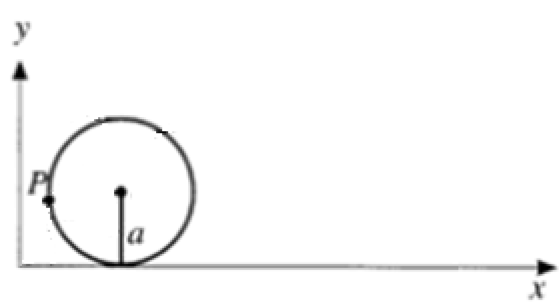
Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:
Let H be the Hamiltonian for a classical system. Show for an arbitrary function f that depends on positions q_i, momenta p_i, and time t, that $\frac{d}{dt}f = \frac{\partial}{\partial t}f + \{f, H\}$ where $\{f, H\}$ indicates the Poisson bracket. Recall that the Poisson bracket $\{a,b\}_{PB} = \sum_k \left(\frac{\partial a}{\partial q_k}\frac{\partial b}{\partial p_k} - \frac{\partial a}{\partial p_k}\frac{\partial b}{\partial q_k}\right)$.

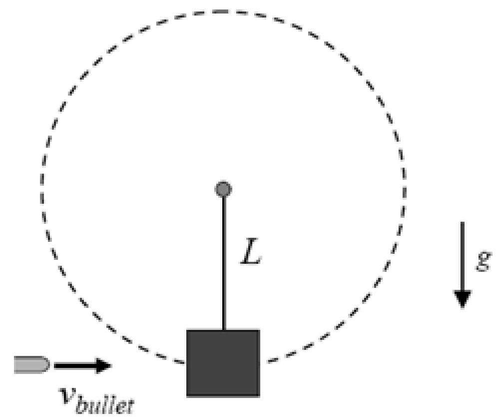
Problem 12:
Suppose the potential energy between an electron and a proton had a term $V_0(a_0/r)^2$ in addition to usual electrostatic potential energy $-e^2/r$, where $e^2 = q_e^2/(4\pi\varepsilon_0)$.
To the first order in V_0, where $V_0 = 0.01$ eV, by how much would the ground state energy of the hydrogen atom be changed?

Problem 13:
A circle of radius *a* rolls on a straight line in the positive x-direction. The trajectory y(x) of a given point P on this circle is a cycloid.



(a) Find the parametric representation x(y) of this cycloid.
(b) Find the length of the path of the point P, when the circle has completed one revolution, i.e. when the center of the circle has traveled a distance $2\pi a$.

Problem 14:
A wooden block of mass M hangs on a massless rope of length L. A bullet of mass m collides with the block and remains inside the block. Find the minimum velocity of the bullet so that the block completes a full circle about the point of suspension.



Problem 15:
An infinitely long straight wire carrying a steady current I, lies along the axis of a linear paramagnetic cylinder of radius R and permeability μ.
(a) Find **H**, **B** and **M** inside and outside the cylinder.
(b) Compute all bound currents flowing in the cylinder.