August 2019 Qualifying Exam

Part II

Mathematical tables are allowed. Formula sheets are provided.

Calculators are allowed.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^8$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

Solve 6 out of the 8 problems! (All problems carry the same weight)

Problem 1:

Consider the one dimensional harmonic oscillator with potential $V(x) = m\omega^2x^2/2$. Use the variational principle to find the ground state energy employing a Gaussian wave function

$\psi(x) = (b/\pi)^{1/4}\exp(-bx^2/2)$,

which is already normalized to 1. "b" is a variational parameter that you must optimize.

(a) Find the kinetic energy $<T>$ in this state.
(b) Find the potential energy $<V>$ in this state.
(c) Optimize "b".
(d) Find the optimal variational energy. How does the result compare with the exact result for a 1D harmonic oscillator?

Problem 2:

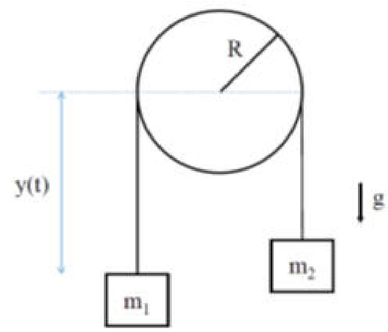
Solve the time independent Schroedinger equation for the eigenfunctions and the corresponding energies of the infinite rectangular well (or "particle in a box") with sides a, b, c. Do not just write down your answer but derive it. Justify each step.

Problem 3:

Find the form of the potential energy for a central force field that allows a particle to move in a spiral orbit given by $r = k\varphi^2$, where k is a constant. What is the total energy of the particle if $U(\infty) = 0$?

Problem 4:

A circular disc of radius R with infinitesimal thickness d is used as a frictionless pulley. The density of the disc varies with the radial distance as $\rho(r) = kr^2$. A massless string of length L is run across it connecting two blocks with masses m_1 and m_2, where $m_1 > m_2$. The string does not stretch and does not slip on the pulley. Treat the blocks as point-like. They only move in the vertical direction.



(a) Find the total mass M of the disc and its moment of inertia around its symmetry axis.
(b) Derive an expression for the kinetic energy of the disc in terms of M and velocity v of the outer edge, assuming the disc rotates at a constant angular velocity.

(c) Provide an expression for a Lagrangian for this system in terms of the vertical distance y(t) measured from the center of the pulley to the block of mass m_1 (see schematics).
Assume that the potential energy vanishes at $y = 0$. Prove that the energy of the system is conserved.
(d) Use the Euler-Lagrangian equation for y(t) to derive an expression for the acceleration d^2y/dt^2, as function of m_1, m_2, M, and g.
(e) Find the tension in the string just above the block m_1 and give a numerical answer, given $m_1 = 10$ kg, $m_2 = 5$ kg, and $M = 30$ kg. Find the tension in the string above block m_2.
(f) What is the torque applied to the pulley?

Problem 5:

Linearly polarized light of the form $E_x(z,t) = E_0 e^{i(kz-\omega t)}$ is incident normally onto a nonmagnetic material which has index of refraction n_R for right-hand circularly polarized light and n_L for left-hand circularly polarized light.
Using Maxwell's equations to calculate the intensity and polarization of the reflected light.

Problem 6:

Assume a spherical planetesimal (very small planet) is orbiting the Sun in a circular orbit with radius R. Assume that the solar luminosity is $3.8*10^{26}$ Watts. Given that for water ice the albedo (reflected intensity/incident intensity) is 0.35 and the sublimation temperature is 200K, find the minimum distance from the Sun that a water ice planetesimal can remain solid. Assume the planetesimal radiates as a black body.

For reference, the Earth orbital radius is $1.5*10^{11}$ m.

Problem 7:

A Λ^0 particle is initially at rest and decays into a proton and a pion, $\Lambda^0 \dashrightarrow p + \pi^-$. The rest masses are $m_\Lambda = 1116$ MeV/c^2, $m_p = 938$ MeV/c^2, and $m_\pi = 140$ MeV/c^2.

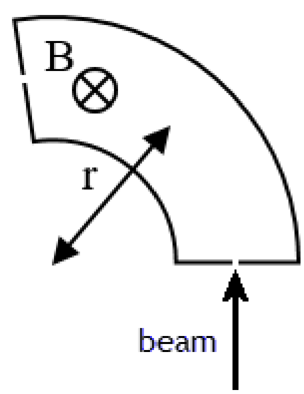
(a) Calculate the total kinetic energy of the decay products.

(b) Find the kinetic energies of the proton and the pion.

(c) Find the speeds of the proton and the pion as fractions of the speed of light c.

Problem 8:

Magnetic separators are typically used in nuclear experimental facilities. The simplest filter consists of a homogenous magnetic field B perpendicular to a fixed curved beamline. Assume a facility is equipped with a variable magnetic field spectrograph of r = 2.8 m bending radius and 75 degrees total turn.



In a nucleon knockout reaction, a single nucleon is removed from a fast moving projectile in a high-energy collision with a light nuclear target. Assume a knockout nuclear reaction on ^{30}Ne results in ^{29}F (Z = 9, M = 29.04 GeV/c^2) or
^{29}Ne (Z = 10, M = 29.02 GeV/c^2)
reaction products with kinetic energy K = 1.2 GeV, which enter the spectrograph.

(a) What is the kinetic energy of the reaction products detected at the exit of the spectrograph?

(b) What is the strength of the magnetic field needed to select ^{29}F with an exit slit at r?

(c) How much longer/shorter does ^{29}Ne take to do the 75-degree turn with the magnetic field selected for ^{29}F?