August 2018 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\lambda)) - 1]^{-1}$

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

A physics graduate student drops a water balloon off a building and hears the splash 3.52 s after letting go of the balloon. How tall is the building, noting that the speed of sound is 331 m/s?

Problem 2:

Estimate the speed of air particles in the room you are in right now.

Problem 3:

The magnetic moment of a Manganese (Mn) atom with magnitude $4\mu_B$ ($\mu_B = 9.3*10^{-24}$ J/T) is initially anti-aligned with and an applied magnetic field of magnitude B = 1 T.
(a) What is the change in energy if the magnetic moment flips and becomes aligned with the magnetic field?
(b) How large must B be to suppress thermal fluctuation of the Mn moment at a temperature of 10 K?

Problem 4:

A metal sphere, of radius R = 20 cm and surrounded by dry air, carries a positive initial charge $Q_i = +10.0$ μC.
(a) Show that, and explain why, this amount of charge is insufficient to ionize the surrounding air. It is known that dry air ionizes when the local electric field strength exceeds approximately $3*10^6$ V/m.
(b) Your body is initially electrically neutral, and the soles of your shoes are made of insulating material. You move your finger toward the sphere without touching the sphere. As your finger gets very close to the sphere (but still not touching it), you observe a spark in the air between your finger and the sphere. Explain why a spark appears, despite Q_i being insufficient to ionize the air before you brought your finger near. A diagram helps clarify your explanation. You can make use of one similar to the one below. Note that there are several elements involved in the explanation. Each element should be clearly identified in your explanation.

(c) After seeing the spark, you move your finger away from the sphere.
The charge on the sphere is now Q_f.
Is Q_f less than, equal to, or greater than Q_i? Explain why, briefly.
(d) In terms of given quantities, what is your net charge, now, quantitatively? Explain why, briefly, in terms of fundamental principles.

Problem 5:

An advanced civilization on a remote spherical planet of radius r_2 has completely excavated and sold its planet's precious core using advanced mining robots. This has resulted in a spherical empty void of radius r_1 at the center of the planet. The density of the massive portion of the planet (extending from a radius of r_1 out to r_2) is ρ. (Be sure to express all your answers in terms of the variables defined in this problem.)

(a) What is the gravitational force on a small robot of mass m when it is located at a radius less than r_1?

(b) In which direction is the force on that robot when it is located at a radius slightly greater than r_1?

(c) At what radius within the massive portion of the planet would the magnitude of the force on that robot be least?

(d) What is the force on that robot when it is located at a radius r between r_1 and r_2?

Problem 6:

An atom has a normalized angular wave function

$\psi(\theta,\varphi) = (1/\sqrt{3})Y_{11}(\theta,\varphi) + (1/\sqrt{6})Y_{21}(\theta,\varphi) + (1/\sqrt{2})Y_{10}(\theta,\varphi)$.

(a) Which values of L^2 can be measured, and with what probability?

(b) What is the expectation value of L_z?

Problem 7:

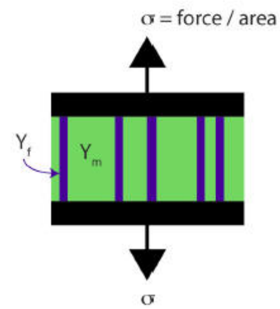
Two distant galaxies A and B are observed by us on the earth to be receding in opposite directions, the speed of recession of each being 0.75c. Will an observer on A be able to see galaxy B? If so what will he say that the speed of recession of B is?

Problem 8:

Consider two identical objects with heat capacity $C_V(T) = \alpha T$ at constant volume, where α is some constant and T is the object's temperature. Suppose one object starts at temperature T_h and is put into contact with the second object that starts at temperature T_c, with $T_h > T_c$. Energy will flow from the hotter object to the colder one. What is the maximum amount of work W_{max} we can extract from this process?

Problem 9:
Determine the energy and the corresponding temperature T of thermal neutrons with the shortest de Broglie wave that can pass through a polycrystalline graphite, whose lattice constant is d = 0.335 nm.

Problem 10:
Two rigid parallel plates are connected by a composite material consisting of parallel fibers in a plastic matrix as shown in the figure. The volume fraction of the fibers relative to the total volume of the composite is f.

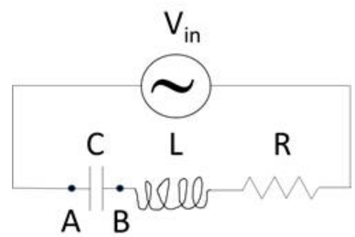


A stress σ is applied to the plates. The Young's modulus of the fiber is Y_f, and that of the matrix is Y_m. Assuming there is no delamination across the fiber/matrix interface, the strains ε in the components of the composite are equal (isostrain).
(a) Write Hooke's law in terms of σ, Y and strain, ε.
(b) Write the expression of the condition of isostrain in terms of σ, Y, ε for the fiber and matrix.
(c) Derive an expression for the Young's modulus, Y_{eff}, of the composite.

Section II:
Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

Consider a series RLC circuit shown below. The circuit is driven by a sinusoidal voltage that has amplitude V_{in}.



(a) At what input frequency is the voltage amplitude between points A and B maximum (assuming a maximum exist)?
(b) For a low damping case determine the voltage amplitude between points A and B at the resonance frequency.
(c) What is the phase shift in this resonant case?
(d) What is the average power dissipated by the circuit (that is by a series connection of R, L and C) at the resonance?

Problem 12:

A rope is lying on the table with ¼ of its length hanging off the table. Find the time it takes for the whole rope to fall and no longer be in contact with the tabletop. Assume that the rope's velocity at $t = 0$ is $v = 0$. The length of the rope is L.
Solve this problem for the case when the coefficient of friction is $\mu = 0$ and $\mu > 0$.

Problem 13:

Calculate the probability current density for of a freely moving particle whose quantum state is described by a de Broglie plane wave.

Problem 14:

A thin film of antireflection coating having refractive index of 1.35 and thickness 452 nm covers surface of a glass plate (n_{glass} = 1.55). The coating is designed so that there is minimal reflectance at normal incidence for 488 nm light. What is the smallest angle of incidence for which this coating produces constructive interference for a reflected 488 nm light beam?

Problem 15:

Tall towers support long, straight, and parallel power lines h = 65 m above the ground and d = 20 m apart carrying f = 60 Hz sinusoidal alternating current, from a power plant to a large city. The amplitude of the current is $I_0 = 3*10^4$ A.

(a) Calculate the amplitude and direction of the magnetic field produced by the two power lines at the base of one of the towers, when the peak current in the left power line in the diagram is flowing out of the page and that in the right line is flowing into the page. Provide both a symbolic and a numeric result (in SI units).

(b) There is concern that this varying field might have different biological effects than the Earth's steady field. For a person lying on the ground near the base of the tower, approximately what is the maximum emf produced around the perimeter of the body, which is about 2 m long by half a meter wide? Again, provide both a symbolic as well as a numeric result.