August 2017 Qualifying Exam

Part II

Mathematical tables are allowed. Formula sheets are provided.

Calculators are allowed.

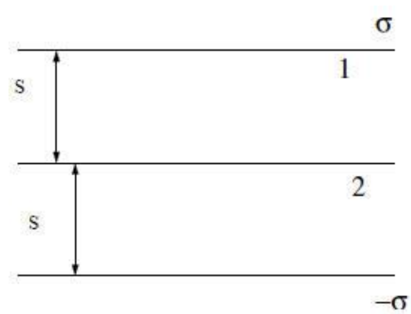
Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

Solve 6 out of the 8 problems! (All problems carry the same weight)

Problem 1:
The space between the plates of a parallel-plate capacitor (see figure) is filled with two slabs of linear dielectric material. Each slab has thickness s, so the total distance between the plates is 2s. Slab 1 has a dielectric constant of 2, and slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is σ and on the bottom plate $-\sigma$.



(a) Find the electric displacement **D** in each slab.
(b) Find the electric field **E** in each slab.
(c) Find the polarization **P** in each slab.
(d) Find the potential difference between the plates.
(e) Find the location and amount of all the bound charge.
(f) Now that you know all the charge (free and bound), recalculate the field in each slab, and compare with your answer to (b).

Problem 2:
Particles evaporating in the z-direction from a hole in a container with an ideal gas at temperature T.
(a) Compute the speed distribution, $f^*(v)$, of the evaporating particles.
(b) Compute the mean z-component of the velocity, $\langle v_z \rangle^*$ of the escaping particles.
(c) Compute the mean energy of the particles leaving the container.
Assume that the equilibrium in the interior of the container is not disturbed by the evaporating particles.

Maxwell-Boltzmann speed distribution: $f(v) = (m/(2\pi kT))^{3/2}\, 4\pi v^2 \exp(-mv^2/(2kT))$

Problem 3:
Consider the following two-state Hamiltonian:
$H = \varepsilon_1|1\rangle\langle 1| + \varepsilon_2|2\rangle\langle 2| + \lambda V(|1\rangle\langle 2| + |2\rangle\langle 1|)$.
For simplicity, let ε_1, ε_2, and λV be positive and let $\varepsilon_1 > \varepsilon_2$.
(a) Calculate the exact eigenvalues and eigenstates.
(b) Calculate the eigenstates to first order and the eigenvalues to second order in λ.

Problem 4:

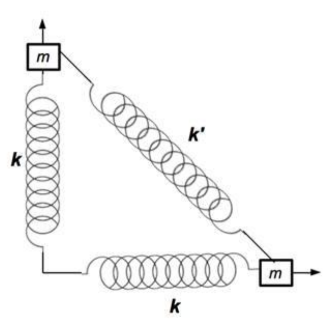
An object is sliding on a frictionless surface with velocity V_0. There is a track on the surface, which turns object backwards as shown by the dashed line in the figure. If the radius of the track is R and the coefficient of kinetic friction between object and track is μ, how long will it take the object to make a 180 degree turn?
The size of the object is negligible compared to the radius of the track.

Problem 5:

Starting with the transformation of the electromagnetic fields under a Lorentz transformation show that
(a) if **E** is normal to **B** in an inertial frame, it must be true in all other inertial frames, and
(b) if $|E| > c|B|$ in an inertial frame, it must be true in all other inertial frames.

Problem 6:

Two equal masses m are constrained to move without friction, one on the positive x-axis and one on the positive y-axis. They are attached to two identical springs of force constant k, whose other ends are attached to the origin. In addition, the two masses are connected to each other by a third spring of force constant k'. The springs are chosen so that the system is in equilibrium with all three springs relaxed (length equal to unstretched length).
Assume small displacements from the equilibrium.
(a) Find the Lagrangian and the equations of motion for the system.
(b) What are the normal frequencies?
(c) Find and describe normal modes.



Problem 7:
A pencil is placed with its point down on a flat surface. Assume that the pencil point is infinitely "sharp" and the surface is perfectly flat. Ignore any extraneous effect such as vibration, air currents etc. The uncertainty principle says that it is not possible to fix the pencil exactly vertically AND exactly at rest. As a result, no matter how well it is positioned, the pencil will fall over in a finite time. Using the uncertainty principle, estimate how long the pencil will take to fall, if it is positioned as well as can be done. Make reasonable estimates for the pencil mass, length, etc.

Problem 8:
Consider the Hamiltonian of a spinless particle of charge q_e in the presence of a static and uniform magnetic field $\mathbf{B} = B\,\mathbf{k}$.
$H = (1/(2m))\,(\mathbf{p} - q_e\mathbf{A}(\mathbf{r},t))^2$.
By using the gauge in which $\nabla\cdot\mathbf{A} = 0$, demonstrate that the Hamiltonian can be expressed as
$H = p^2/(2m) - (q_e/(2m))\,\mathbf{L}\cdot\mathbf{B} + (q_e^2B^2/(8m))(x^2 + y^2)$.
Note that the second term corresponds to the linear coupling between the external field and the magnetic moment.