August 2017 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

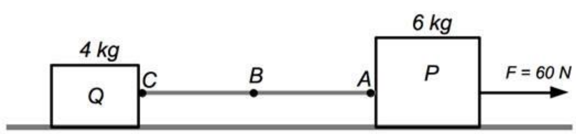
Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

Useful integral:
$\int \sin^2(x)dx = x/2 - \sin(2x)/4$

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:
Two blocks P and Q of masses 6 kg and 4 kg, respectively, are connected through a rope of mass 2 kg. The two masses are placed on a frictionless platform as shown in the figure. The system of the blocks is pulled in the forward direction by a force of 60 N applied on the block P.



(a) Find the acceleration of the system?
(b) Find the tension in the rope at points A, B and C?

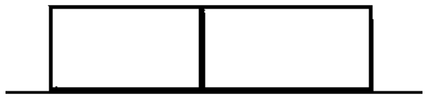
Problem 2:
A charged capacitor with C = 590 μF is connected in series to an inductor that has L = 0.330 H and negligible resistance. At an instant when the current in the inductor is I = 2.50 A, the current is increasing at a rate of dI/dt = 73.0 A/s. During the current oscillations, what is the maximum voltage across the capacitor?

Problem 3:
A thin lens creates an image of an object which is 4 times larger than the object. The object is then moved to a new location on the same side of the lens and the image is again 4 times larger than the object.
(a) Is the lens converging or diverging?
(b) What is the ratio of the distance the object has been moved to the focal length of the lens?

Problem 4:
The kinetic energy of a non-relativistic mass point equals $T = p^2/(2m)$, with $\mathbf{p} = m\mathbf{v}$ the momentum of the particle. Find a formally similar expression for the relativistic kinetic energy in terms of the relativistic momentum $\mathbf{p} = \gamma m\mathbf{v}$.

Problem 5:

An insulated container of mass M and length L is at rest on a horizontal frictionless surface. The container is filled with an ideal gas of unknown mass m and temperature T. The container is divided in half by a light movable insulated piston.

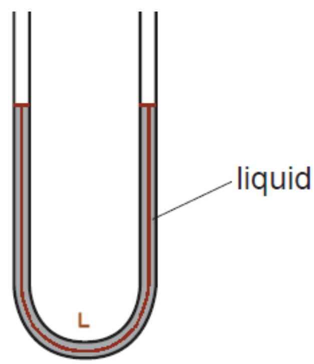


A heater is turned on inside the left part of the container, bringing the temperature of the gas there to 2T. The temperature of the gas inside the right part of the container remains unchanged. As a result, the container moves a distance x along the surface. Find the mass of the gas m in terms of L and M.

Problem 6:

A "U-Tube" with a circular cross section of radius r is partially filled with a liquid of density ρ. The total length of filled tube is L. In equilibrium, the height of the two free surfaces at the top of the "U" will be the same. If this equilibrium is disturbed by causing one surface to go up and the other down (conserving fluid), the fluid will begin to oscillate. What is the frequency of this oscillation?

Assume that the liquid has zero viscosity and that the local gravitational acceleration is g.



Problem 7:

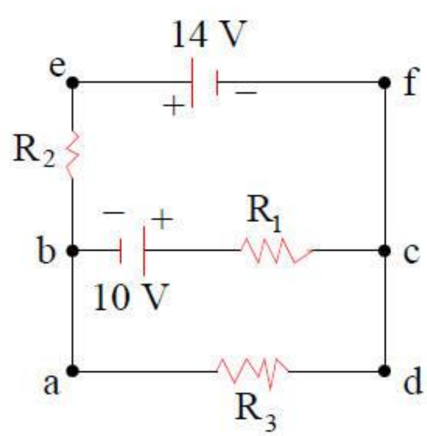
In the circuit shown in the figure R_1 = 6 Ω, R_2 = 4 Ω, and R_3 = 2Ω.

(a) Find the currents I_1, through resistor R_1, I_2, through resistor R_2, and I_3, through resistor R_3, in the circuit shown in the figure.

(b) Indicate in what direction each current flows. Give the results in terms of the junctions labeled in the circuit.

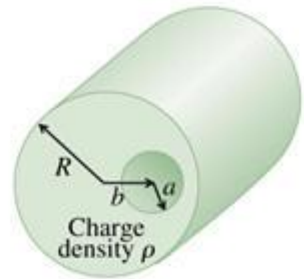
For example, you need to say whether I_1 goes from b to c or from c to b.

(c) Find the potential difference between junctions b and c. Clearly indicate which junction has the higher potential.



Problem 8: JL

A very long, solid insulating cylinder with radius R has a cylindrical hole with radius a bored along its entire length. The axis of the hole is a distance b from the axis of the cylinder, where a < b < R. The solid material of the cylinder has a uniform volume charge density ρ. Find the magnitude and direction of the electric field inside the hole.

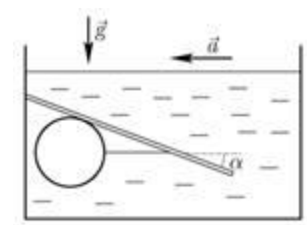


Problem 9: GG

A particle of mass m is in the ground state of an infinite square well
($U = 0$ for $0 < x < a$ and $U = \infty$ otherwise).
At time $t = 0$, the right "wall" (i.e. at $x = a$) shifts to $x = 2a$. A measurement of the energy of the particle is made just after the wall shifts. (Assume that all this happens so quickly so the spatial wave function of the particle does not change). What is the probability that the energy measurement yield a value EXACTLY the same as the energy of the ground state of original well?

Problem 10: YE

A hollow ball with a volume V is held in place in a tank under water by a wire under a sloped plank as shown in the figure. The water density is ρ and average ball density is ρ/5. The plank makes an angle α with the horizontal, with $\tan(\alpha) = 1/3$.
What is the tension in the wire if the whole system is accelerating horizontally towards the left with acceleration $a = g/6$.

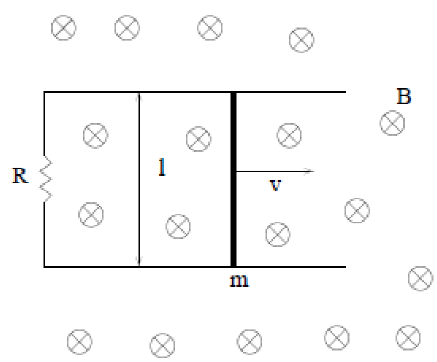


Section II:
Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:
A particle of mass m moves in a one-dimensional potential given by
$V(x) = -W$ for $|x| < a$, $V(x) = 0$ for $|x| \geq a$.
Demonstrate that this potential has at least one even bound state.

Problem 12:
From the Hamiltonian of a free, non-relativistic, classical mass-point, $H = \mathbf{p}^2/(2m_0)$,
(a) find the equation of motion $d\mathbf{p}/dt$ and $d\mathbf{q}/dt$.
(b) Compute the total differential for an arbitrary function $F(\mathbf{p}, \mathbf{q}\ t)$ and express in terms of Poisson brackets.

Problem 13:
A metal bar of mass m slides without friction on two parallel conducting rails a distance l apart (see figure). A resistor R is connected across the rails and a uniform magnetic field **B**, pointing into the page, fills the entire region.



(a) If the bar moves to the right at speed v, what is the current in the resistor? Indicate also in what direction the current flows.
(b) What is the magnetic force on the bar? Indicate the direction of the force and provide the result in terms of the data given.
(c) If the bar starts out with speed v_0 at time $t = 0$, and is left to slide, what is its speed at a later time t?
(d) The initial kinetic energy of the bar was, of course, $mv_0^2/2$. Where does this energy go? Prove that energy is conserved in this process by showing that the energy gained elsewhere is exactly $mv_0^2/2$.

Problem 14:

A stationary space station can be approximated as a hollow spherical shell of mass 6 tons (6000 kg) and inner and outer radii of 5 m and 6 m. To change its orientation, a uniform fly wheel of radius 10 cm and mass 10 kg located at the center of the station is spun quickly from rest to 1000 rpm.

(a) How long (in minutes) will it take the station to rotate by 10^o?

(b) What energy (in Joules) is needed for the whole operation?

(Moment of Inertia of hollow spherical shell: $[2M(R_o^5 - R_i^5)]/[5(R_o^3 - R_i^3)]$ where R_o is outer radius and R_i is inner radius)

Problem 15:

For a certain system, the operator corresponding to the physical quantity A does not commute with the Hamiltonian. It has eigenvalues a_1 and a_2, corresponding to the eigenfunctions $\Phi_1 = 2^{-1/2}(u_1 + u_2)$ and $\Phi_2 = 2^{-1/2}(u_1 - u_2)$, respectively, where u_1 and u_2 are eigenvalues of the Hamiltonian with eigenvalues E_1 and E_2. If the system is in the state $\psi = \Phi_1$ at time $t = 0$, calculate the time evolution of the expectation value of A.