January 2024 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\,400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\,7685 * 10^{-3}$ m K
Planck radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\,\lambda)) - 1]^{-1}$

Useful integrals:

$\int_0^{\pi} \sin^4(x)\,dx = 3\pi/8$

$\int_0^{\pi} \sin^2(x)\sin^2(2x)\,dx = \pi/4$

Section I:

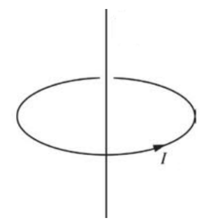
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

A projectile is shot upward from the surface of the Earth at half the escape velocity. If the escape velocity of the Earth is 11.2 kilometers per second and the radius of the Earth is 6356 km, what is the maximum height reached by the projectile?

Problem 2:

Consider a ring of radius R with a circulating current I.
Using the Biot-Savart law, find the magnetic field **B** induced by this current along the axis that is perpendicular to the ring and goes through its center.
From the positive side of this axis, the current circulates anticlockwise.



Problem 3:

Two objects are dropped from the top of a cliff of height H. The second object is dropped when the first has traveled a distance D. At the instant when the first object reaches the bottom, what is the distance of the second object above the ground in terms of H and D?

Problem 4:

For a classroom demonstration, you pass the beam from a HeNe laser (λ = 633 nm) through two identical, closely-spaced slits and observe the interference pattern on a wall 5 m away and perpendicular to the plane containing the slits. In the darkened classroom, you can clearly observe up to 10 interference maxima on both sides of the central maximum, but the 4^{th} and 8^{th} maxima are missing on either side. The spacing between adjacent maxima on the wall is 1.1 cm.

(a) What is the spacing between the slits?

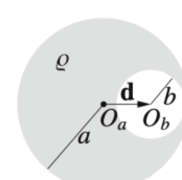
(b) What is the slit width?

Problem 5:

In reference frame S a firecracker explodes and a second firecracker explodes a distance Δx = c*25 ns away and 52 ns later. In another inertial reference frame S', moving with velocity v **i** with respect to S, the two explosions are measured to occur a distance d' = c*42 ns apart in space. How much time passes between the explosions in frame S'?

Problem 6:

A sphere of radius a has uniform charge density ϱ over all its volume, excluding a spherical cavity of radius where b < a where ϱ = 0. The center of the cavity, O_b, is located at a distance d from the center of the sphere, O_a, with d + b < a. Find the electric field inside the cavity.



Problem 7:

(a) Consider three bosons, without spin, in a one-dimensional infinite square well with energy levels $E_n = n^2E_1$, n = 1, 2, 3, … . What is the energy of the ground state?

(b) Repeat for three fermions, assumed without spin.

Problem 8:

Why are there tides? Describe the mechanism in a few sentences, including what circumstances lead to maximally high and low tides.

Problem 9:

Two party balloon have a volume of 0.03 m^3 each. One is filled with air at 1.1 atmospheric pressure and its mass, including skin is 40.6 g. The other is filled with helium at 1.1 atmospheric pressure and its mass, including skin, is 6.8 g. The density of the air is 1.2 kg/m^3.

A science museum has built an "elevator" which consists of a chamber with scales at the top and the bottom that can read the forces pushing against or pulling on the floor or the ceiling of the cart. At the push of a button, the cart accelerates at a rate of 2 m/s^2 upward for 2 s, and then it decelerate at the same rate until it stops at its maximum height.

The air balloon is suspended from the ceiling with a string of negligible mass, and the helium balloon is fixed to the floor with the same type of string.

(a) What is the maximum height reached by the bottom of the cart?

(b) What is the tension in the strings when the cart is at rest, and in which direction do the strings pull on each balloon?

(c) What is the tension in the strings while the car is accelerating upward, and in which direction do the strings pull on each balloon?

Let g = 10 m/s^2. Assume the density of the air is constant in the chamber.

Problem 10:

A man of mass 80 kg achieves ≈ 50 m/s in free fall in the air. A parachutist with the same mass reaches 5 m/s. Assume that the air resistance force is proportional to the velocity $\mathbf{F} = -k\mathbf{v}$ and assume the buoyant force can be neglected.

(a) What are the values of the k in both of these cases?

(b) For both cases, what are the distances traveled in t = 10 s if the initial speed is zero?

Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

A beam of muons is accelerated such that each muon has a total energy of 5 TeV (1 TeV = 10^{12} eV).

(a) If the muon lifetime is 2.2 μs and the muon mass is 106 MeV/c^2, what is the velocity of the muon beam?

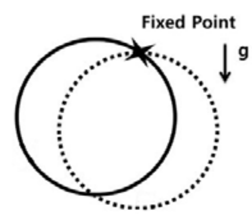
(b) What will be the average decay time of a muon as measured in the lab frame?

(c) If two such muon beams are traveling in a circular collider with circumference 15 km, how long will it take each muon to make a full trip around the ring?

(d) If the two muon beams are made to cross and produce particle collisions once per turn, how many times will the average muon go through this crossing before decaying?

Problem 12:

A hoop of mass m and radius R = 1.25 m can oscillate in the vertical plane about a fixed point.



(a) What is the equation of motion for this physical pendulum for small angular displacements θ from equilibrium?

(b) What is the angular frequency ω for small oscillations?

(c) If the pendulum is released from rest at time t = 0 at θ = 3°, solve for θ(t). Let g = 10 m/s^2.

Problem 13:

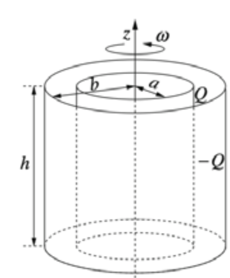
Two identical spin zero bosons are placed in a one-dimensional infinite square well, U(x) = ∞, x < 0 and x > a, U(x) = 0, 0 < x < a. The bosons interact weakly with one another via the potential energy function $U(x_1,x_2) = -aU_0\delta(x_1 - x_2)$, where U_0 is a constant with the dimensions of energy.

(a) First, ignoring the interaction between the particles, find the ground state and first excited state wave functions and the associated energies.

(b) Use first-order perturbation theory to estimate the effect of the particle-particle interaction on the energies of the ground state and the first excited state.

Problem 14:

The concentric cylindrical shells of a cylindrical capacitor have radii a and b > a, respectively, and height h >> b. The capacitor charge is Q, with +Q on the inner shell of radius a, and Q on the outer shell of radius b (see figure). The whole capacitor rotates about its axis with angular velocity ω = 2π/T. Neglect edge effects.



(a) Find the capacitance of the capacitor.

(b) Evaluate the magnetic field B generated by the rotating capacitor over all space.

(c) Find the direction and magnitude of the Poynting vector.

Problem 15:

You prepare an ensemble of identical experiments at $t = 0$. You measure the values of a physical observable A at time $t = t_1 > 0$ in all of the experiments of the ensemble. At time $t_2 > t_1$ you measure observable A again in all of the experiments – i.e., you follow the measurement at $t = t_1$ in each experiment with a subsequent measurement of A in each experiment at $t = t_2$.

True or False: (Explain your choice in one or two sentences.)

(a) The values you obtain for A across all of the experiments at time $t = t_1$ are the same given you have made all of the measurements at the same time $t = t_1$.

(b) The value you obtain for A in each experiment at $t = t_2$ may be different relative to the value obtained for A in the same experiment at $t = t_1$.

(c) The expectation value of A is the same at both $t = t_1$ and $t = t_2$.

(d) The values you obtain for A in any of the experiments at either $t = t_1$ or $t = t_2$ must be one of the eigenvalues of its associated Hermitian operator.

At $t = t_3 > t_2$ you measure a second physical observable B across all of the experiments – i.e., you follow the measurement of A at $t = t_2$ in each experiment with a measurement of B at $t = t_3$ in each experiment. The commutator of the two Hermitian operators associated with A and B satisfies $[A, B] \neq 0$.

True or False:

(e) The values you obtained for A at $t = t_2$ in any of these experiments will remain the same at $t = t_3$ given you are measuring a different physical quantity B, not A.

Now you reset all of the experiments to what they were at $t = 0$, i.e., you start all over again with the same initial set of identical experiments. At time $t = t_4$, you measure A again in all of your experiments. Moreover, the commutator of the operator associated with A and the Hamiltonian operator satisfies $[H, A] = 0$.

True or False:

(f) The value you obtain for A in each experiment at $t = t_4$ is the same as the value you obtained for A in the corresponding experiment at $t = t_1$.

(g) The expectation values of A at $t = t$ and $t = t_4$ are the same.