January 2022 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Planck radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\,\lambda)) - 1]^{-1}$

Useful integrals:

$$\int \sin^2 x\, dx = \frac{1}{2}x - \frac{1}{4}\sin(2x)$$

$$\int x\, e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1), \quad \int e^{ax} dx = \frac{e^{ax}}{a}$$

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:
A box slides down an inclined plane. At some time its speed is measured to be v = 2 m/s, and it slides an additional distance d = 2 m before it comes to a stop. If the incline makes an angle of $\theta = 20^o$ with the horizontal, determine the coefficient of kinetic friction μ_k.

Problem 2:
A spherical, shiny decoration ball is acting as a mirror. The sphere has a radius of 10 cm.
Your eye is 30 cm from the mirror surface. How much bigger or smaller is the image of your eye than the actual size of your eye? Is the image real or virtual, upright or inverted?

Problem 3:
Suppose a particle of mass m, initially at rest, is hit by a photon. The photon has energy mc^2 and is completely absorbed by the particle. What is the rest mass of the particle after the collision?

Problem 4:
What is the diameter of the largest asteroid from which you can escape by jumping? Make reasonable approximations to keep your calculations simple. Explain these approximations clearly. You can take the specific gravity (i.e. density relative to water) of the asteroid to be 3.

Problem 5:
A beam of electrons enters a magnetic field of B = 2 T. The field is perpendicular to the electron beam. You measure the radius of the electron orbit to be R = 1 mm. What is the velocity of the electrons relative to the speed of light, c?

Problem 6:
Consider a particle in one-dimension in a quantum state described by the wave function

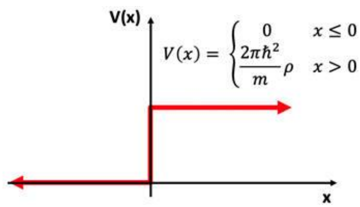
$\psi(x) = (3/a^3)^{1/2}(a - x)$ for $0 \leq x \leq a$,
$\psi(x) = 0$ everywhere else.

(a) Where is the most probable place to find the particle?
(b) What is the expected value for the position $\langle x \rangle$?

Problem 7:
A physics student has a side gig playing guitar in a band. They observe that when a certain tension was applied to a particular string, it stretched by 1%. What is the speed of a transverse wave travelling along this string when it is under this tension? The volume density of the string material is $\rho = 8*10^3$ kg/m^3 and its Young's modulus is $2*10^{11}$ N/m^2. Express your answer in m/s.

(Young's modulus is the ratio of the force per unit area F/A, called the stress, to the fractional change in length ΔL/L, called the strain.)

Problem 8:
A particle with mass m and wave vector k_0 is incident upon a potential step at the origin, as the particle travels from left to right.



(a) Derive an expression for the wave vector of the particle for x > 0 in terms of k_0 and ρ.
(b) Using (a) what is the condition for total reflection of the incident wave function in terms of k_0 and ρ?

Problem 9:
A coaxial cable is a type of electrical cable consisting of an inner conductor surrounded by a concentric conducting shield, with the two separated by an insulator. Consider a very long, straight coaxial cable. Suppose the inner cylindrical conductor has radius R_1, and the shield has inner radius R_2 and outer radius R_3. They carry the same current I but along opposite directions, and the current is uniformly distributed within their cross sections. We know the relative permeability of the conductors μ_1 and the relative permeability of the insulator μ_2. Find the magnetic field **B** inside and outside near the cable, away from the ends.

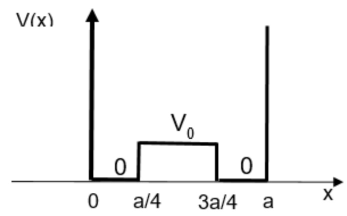
Problem 10:
Two large parallel metal plates of area A are separated by a distance d, small compared to the size of the plates. The distance d is small enough that the plates can be treated as if they were infinite in extent. A battery maintains a potential difference V across the plates. An external agent slowly exerts an uncharged conducting slab of the same area A and thickness d/2 is midway between the two plates, not touching either of them. After the slab has been inserted, it is held in place at rest.
(a) What is the capacitance of the configuration before and after the slab is inserted?
(b) How much work was done by the external agent inserting the slab?

Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

Using perturbation theory to first order, find the corrected energy of the ground state (n = 1) of the "modified" infinite square well shown in the figure.



Problem 12:

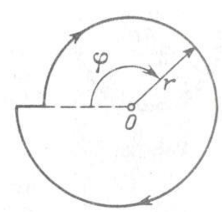
An unstable particle is moving with velocity v' along the y' axis in reference frame K'. Frame K' is moving relative to frame K along the x-axis with velocity V in the positive x-direction. The axes of the coordinate systems in K' and K are parallel to each other and the x-axes are aligned. If the particles mean life in its rest frame is t_0, find the mean distance it travels in K.

Problem 13:

Derive Wien's displacement law from the Planck radiation law expressed in terms of wavelength. Estimate the wavelength maximum for a temperature $T = 10^4$ K.

Problem 14:

A current I flows as shown in the plot, where $r = r_0(1 + 0.05\ \varphi)$, with $0 \leq \varphi \leq 2\pi$. Find magnetic field **B** at the point O.



Problem 15:

A rarefied gas consists of identical dipole molecules with dipole moments **p**. The gas with concentration n = N/V is in statistical equilibrium at temperature T in a uniform electric field **E** = E**k.**

Neglect the interaction of molecules with each other and the deformation of their electron shells.

(a) What is the range of energies for each dipole in the electric field?

(b) Find the polarization vector **P** (the dipole moment per unit volume) of the gas as a function of the magnitude of the electric field;