August 2021 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\,400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\,7685 * 10^{-3}$ m K
Planck radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\,\lambda)) - 1]^{-1}$

Section I:

Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

An astronaut on the space station lets go of a flashlight whose mass is 1 kg and which is initially at rest with respect to the astronaut. If the light output is 1 W and the battery lasts for 1 hour, estimate the final velocity of the flashlight relative to the astronaut.

Problem 2:

Suppose we are observing a planet moving around a star (from the rest frame of the star) with constant speed v in a circular orbit of radius R. Suppose the speed v is fast enough that relativistic effects are important. Calculate the proper time for the planet to orbit the star (the length of a year on the planet) in terms of v and R.

Problem 3:

A box with mass m slides down a 30 m frictionless plank, angled at 30 degrees. It starts from rest at the top. When it has moved 10 m down the plank, a mass m' = ¼ m drops into the box from above and sticks to the inside bottom of the box. What is the speed of the box when it has moved 25 m from the top?

Problem 4:

One of the most prominent spectral lines of hydrogen is the H_α line, a bright red line with a wavelength of 656.1 nm. What is the wavelength of the H_α line emitted from a star receding from the observer with a speed of 3000 km/s?

Problem 5:

Two singly charged ions with charge of opposite sign and masses m_1 and m_2 rotate around their common center of mass. The size of ions is negligible compared to their separation. This pair of ions is in thermal equilibrium with a monatomic gas at T = 3000 K. What is the magnitude of the average electric dipole moment of this pair of ions? Give a numerical answer.

Problem 6:

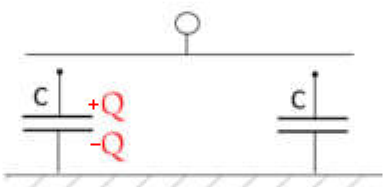
Consider the position X and momentum P operators in a one-dimensional quantum system. Calculate the commutator $[X, P^3]$.

Problem 7:
A beam of non-relativistic protons passes without deflection through a region with uniform crossed electric and magnetic fields. The fields are perpendicular to each other and have a field strength of E = 120 kV/m and B = 50 mT. The beam is stopped by a grounded target. What is the force the beam exerts on the target if the current is 0.8 mA?

Problem 8:
Consider a 3-state quantum mechanical system with the Hamiltonian $H = \varepsilon\begin{pmatrix} 2 & 0 & 0.1 \\ 0 & 3 & 0 \\ 0.1 & 0 & 2 \end{pmatrix}$.

Estimate the eigenvalues of this Hamiltonian using perturbation theory. Do you have to use degenerate or non-degenerate perturbation theory?

Problem 9:
Assume you have two equal capacitors, arranged as shown. Initially the plates of the left capacitor hold charge +Q and -Q, respectively, and the left capacitor is not charged.



(a) What is the energy stored in the left capacitor?
(b) The capacitors are now connected by a conducting rod and the charge redistributes.
What is total the energy stored in both capacitors now? Explain the difference. Should energy not be conserved?

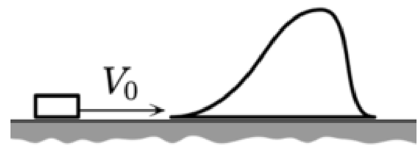
Problem 10:
(a) Some transmission electron microscopes accelerate electrons across a 1 MV potential difference. What is the wavelength of a 1 MeV electron?
(b) A neutron moderator typically consists of hydrogenous material, e.g., liquid hydrogen (or deuterium). Multiple interactions with hydrogen reduce the MeV energy of a neutron, resulting in neutrons with energies of order meV. What is the wavelength of a 4 meV neutron?

Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

A small puck of mass m is heading toward a slide at a speed v_0.
The slide of mass 3m rests on a frictionless horizontal table. Then the puck climbs the slide without friction while remaining in contact with it, reaches a high point, and then reverses direction.

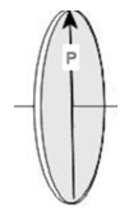


(a) What is the final speed of the puck in terms v_0?

(b) What is the maximum height the puck will reach before reversing direction?

Problem 12:

A uniform dielectric round plate has radius R and thickness d (R >> d). It is uniformly polarized with the polarization **P** parallel to the plate. Find the electric field generated by the polarization at the center position of the plate.



Problem 12:

The resistivity of a 99.999% pure gold wire decreases by two orders of magnitude as the temperature is reduced from 900 ^oC to 523 ^oC. You are told the resistivity is proportional to the equilibrium concentration of vacancies. Treat the wire as a two-state system. Use the Boltzmann statistical formula to relate the number of vacancies, n_v, to the number of atoms, N, in terms of the energy of vacancy formation, E_v, and temperature, T.

(a) Write down the formula relating the number of vacancies, n_v, to the number of atoms, N.

(b) Calculate the energy of vacancy formation, E_v.

(c) Calculate the equilibrium vacancy concentration per N atoms at the melting point of gold (T_m = 1065 ^oC).

Problem 14:

An ideal diatomic gas of volume $2.00*10^{-3}$ m^3 at 300 K and atmospheric pressure ($P_{atm} = 1.13*10^5$ Pa) is compressed isobarically to 1/5 of its original volume.

(a) How many moles of gas are present? Recall the value of the gas constant: R = 8.314 J/(mol K).

(b) How much work is done on the gas?

(c) What is its new temperature?

(d) What is the change in its internal energy?

(e) How much heat flows into (+) or out of (−) the gas?

(f) The gas now expands isothermally to its original volume. How much work is done by the gas?

(g) What is the final pressure after the process in part (f)

Problem 15:

A spin one-half particle is in an eigenstate of $S_n = \mathbf{S}\cdot\mathbf{n}$ with eigenvalue $\hbar/2$. **S** is the spin operator, and **n** is a unit vector within the xz plane, pointing away from the positive z-direction by an angle θ.

(a) What is the probability of obtaining $\hbar/2$ from measuring S_x?

(b) What is the uncertainty ΔS_x of S_x?