August 2020 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^8$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

Section I:

Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

Find the energy necessary to move a charge Q from point A = (a,0,0) to point B = (a,0,h), where a and h are positive constants, along a helical path parametrized as

$\mathbf{r} = a \cos\theta\, \mathbf{i} + a \sin\theta\, \mathbf{j} + h\theta/(2\pi)\, \mathbf{k}$,

under the influence of an electrostatic field $\mathbf{E} = -E_0\, \mathbf{k}$.

Problem 2:

A rugby player runs with the ball directly towards his opponent's goal, along the positive direction of an x axis. He can legally pass the ball to a teammate as long as the ball's velocity relative to the field does not have a positive x component. Suppose the player runs at speed 4.0 m/s relative to the field while he passes the ball with a speed of 6.0 m/s relative to himself. What is the smallest angle (relative to the x axis) the ball can be passed in (as seen from the player) in order for the pass to be legal?

Problem 3:

A particle in some unspecified potential is in the initial normalized state

$\psi(x) = a\, x\, e^{-x/b}$, $x \geq 0$,
$\psi(x) = 0$, $x < 0$.

(a) Where are you most likely to find the particle if you were to measure its position?
(b) What is the probability of finding the particle in the region $x > 0$?

Problem 4:

A small air bubble of initial radius r = 1 cm is introduced at the bottom of a lake that is 20 m deep. The bubble expands as it rises slowly. Assume the lake has the same temperature everywhere.
(a) What is the pressure at the bottom of the lake?
(b) What is the radius of the bubble after it rises to the surface?

Problem 5:

Consider the vector field $\mathbf{E} = c(2x^2 - 2xy - 2y^2)\mathbf{i} + c(y^2 - 4xy - x^2)\mathbf{j}$, where c is a constant, in a certain volume V of space.
(a) Verify that this field represents an electrostatic field.
(b) Determine the volume charge density in the volume V consistent with such a field.

Problem 6:

You need to determine the density of a ceramic statue. If you suspend it from a spring scale, the scale reads 28.4 N. If you then lower the statue into a tub of water, so that it is completely submerged, the scale reads 17.0 N. What is the statue's density?

Problem 7:

Estimate (roughly) the net rate of heat loss by your body due to radiation when you are in a room with a temperature of 10 ^{o}C.

Problem 8:

An operator $\underline{A}$, representing observable A, has two normalized eigenstates Ψ_1 and Ψ_2, with eigenvalues a_1 and a_2, respectively. An operator $\underline{B}$, representing observable B, has two normalized eigenstates Φ_1 and Φ_2, with eigenvalues b_1 and b_2, respectively. The eigenstates are related by

$\Psi_1 = (3\Phi_1 + 4\Phi_2)/5$,

$\Psi_2 = (4\Phi_1 - 3\Phi_2)/5$.

(a) Observable A is measured, and the value a_1 is obtained. What is the state of the system immediately after this measurement?

(b) If B is now measured immediately following the A measurement, what are the possible results, and what is the probability of measuring each result?

(c) Right after the measurement of B, A is measured again. What is the probability of getting a_1?

Problem 9:

Two-dimension polar coordinates have basis vector $\mathbf{e}_r$ and $\mathbf{e}_\phi$.

(a) Draw a figure that shows the Cartesian basis vectors **i** and **j**, polar basis vectors $\mathbf{e}_r$ and $\mathbf{e}_\phi$ for an arbitrary position vector in the first quadrant.

(b) Recall that the Cartesian infinitesimal line element is $d\mathbf{r} = dx\,\mathbf{i} + dy\,\mathbf{j}$.

Express the infinitesimal line element in terms $\mathbf{e}_r$ and $\mathbf{e}_\phi$.

(c) Derive an expression for the velocity **v** in terms of polar basis vectors $\mathbf{e}_r$ and $\mathbf{e}_\phi$.

Problem 10:

(a) What are the electromagnetic fields corresponding to the electromagnetic potentials

(i) $\Phi = 0$; $\mathbf{A} = (0, xB_0, 0)$ and

(ii) $\Phi = 0$; $\mathbf{A} = (-yB_0, 0, 0)$?

(b) How are these potentials related?

(c) How many degrees of freedom are needed to describe electromagnetic fields?

(d) How many Maxwell Equations are there? Explain with view on part (c).

Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

Consider a spin ½ particle in the presence of a uniform static magnetic field **B** = B_0**i**. Suppose that at t = 0 the spin state of the particle is the $|-\rangle_z$ eigenket of S_z.

(a) State briefly how you would prepare such a state.

(b) Suppose at time t > 0 we measure the z-component of the spin. What values can be obtained and with what probabilities?

(c) Evaluate the mean value of that measurement and comment on the physics of your result.

Problem 12:

Find the speed of an electron that has been accelerated from rest by an electric field through a potential increase of

(a) 20.0 kV and

(b) 5.00 MV, typical of a high-voltage x-ray machine.

Give your answers in units of the speed of light.

Problem 13:

A particle with mass m is in the ground state of an infinite square well of length L.

(a) What is the probability to measure the particle at the center of the well within an uncertainty Δ, where Δ << L?

(b) What is the probability that a subsequent measurement of the energy will yield a result that is different from the ground-state energy?

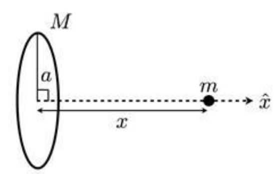
(c) Will the particle's state after this last measurement be more likely an even or odd wave function (with respect to reflection at the well's center), or is it impossible to tell? Why?

Problem 14:

Consider a pendulum in a plane (i.e. a "2D world"), consisting of a mass m attached at the end of a weightless rope of length l_0. When the pendulum is set into motion the length of the rope is shortened at a constant rate dl/dt = -α = constant. Compute the Lagrangian, write down the equation of motion, and discuss the conservation of energy for this system. Does the sign of α matter for energy conservation?

Problem 15:

Consider the ring of mass M shown in the figure. Assume that the ring is thin and has a uniform density. A particle of mass m is placed a distance x from the center of the ring, on a line that passes through the center of the ring and is perpendicular to its plane.



(a) Calculate the gravitational potential energy of this system, assuming that the potential is zero at infinity.
(b) Calculate the gravitational force acting on the particle.
(c) Derive expressions for gravitational potential energy and force in the limit x >> a.
(d) Why would you expect the result in (c)?