Problem 1:

Consider a set of wave functions $\psi_i(x)$. Which of the following conditions guarantees that the functions are normalized and mutually orthogonal? (The indices i and j take on the values in the set $\{1, 2, \ldots, n\}$.)

(A) $\psi_i^*(x)\psi_j(x) = 0$

(B) $\psi_i^*(x)\psi_j(x) = 1$

(C) $\int_{-\infty}^{\infty} \psi_i^*(x)\psi_j(x)\,dx = 0$

(D) $\int_{-\infty}^{\infty} \psi_i^*(x)\psi_j(x)\,dx = 1$

(E) $\int_{-\infty}^{\infty} \psi_i^*(x)\psi_j(x)\,dx = \delta_{ij}$

Problem 2:

A muon can be considered to be a heavy electron with a mass $m_\mu = 207m_e$. Imagine replacing the electron in a hydrogen atom with a muon. What are the energy levels E_n for this new form of hydrogen in terms of the binding energy of ordinary hydrogen E_0, the mass of the proton m_p, and the principal quantum number n?

(A) $E_n = \frac{-E_0}{n^2}\left(\frac{m_\mu}{m_e}\right)$

(B) $E_n = \frac{-E_0}{n^2}\left(\frac{m_e}{m_\mu}\right)$

(C) $E_n = \frac{-E_0}{n^2}\left(\frac{(m_p + m_e)}{(m_p + m_\mu)}\right)$

(D) $E_n = \frac{-E_0}{n^2}\left(\frac{m_\mu(m_p + m_e)}{m_e(m_p + m_\mu)}\right)$

(E) $E_n = \frac{-E_0}{n^2}\left(\frac{m_e(m_p + m_\mu)}{m_\mu(m_p + m_e)}\right)$

Problem 3:

A particle can occupy two possible states with energies E_1 and E_2, where $E_2 > E_1$. At temperature T, the probability of finding the particle in state 2 is given by which of the following?

(A) $\frac{e^{-E_1/kT}}{e^{-E_1/kT} + e^{-E_2/kT}}$

(B) $\frac{e^{-E_2/kT}}{e^{-E_1/kT} + e^{-E_2/kT}}$

(C) $\frac{e^{-(E_1+E_2)/kT}}{e^{-E_1/kT} + e^{-E_2/kT}}$

(D) $\frac{e^{-E_1/kT} + e^{-E_2/kT}}{e^{-E_2/kT}}$

(E) $\frac{e^{-E_1/kT} + e^{-E_2/kT}}{e^{-E_1/kT}}$

Problem 4:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Consider the Pauli spin matrices σ_x, σ_y, and σ_z and the identity matrix I given above. The commutator $[\sigma_x, \sigma_y] \equiv \sigma_x\sigma_y - \sigma_y\sigma_x$ is equal to which of the following?

(A) I (B) $2i\sigma_x$ (C) $2i\sigma_y$ (D) $2i\sigma_z$ (E) 0

Problem 5:

An electron with total energy E in the region $x < 0$ is moving in the $+x$-direction. It encounters a step potential at $x = 0$. The wave function for $x \le 0$ is given by

$$\psi = Ae^{ik_1 x} + Be^{-ik_1 x}, \text{ where } k_1 = \sqrt{\frac{2mE}{\hbar^2}};$$

and the wave function for $x > 0$ is given by

$$\psi = Ce^{ik_2 x}, \text{ where } k_2 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}.$$

Which of the following gives the reflection coefficient for the system?

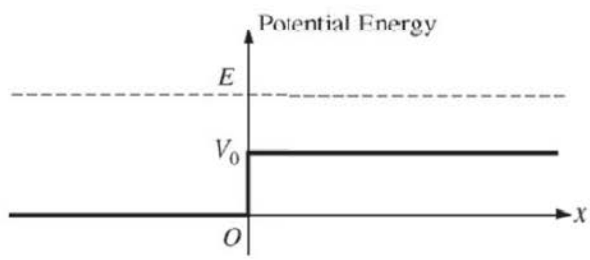
(A) $R = 0$

(B) $R = 1$

(C) $R = \frac{k_2}{k_1}$

(D) $R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$

(E) $R = \frac{4k_1k_2}{(k_1 + k_2)^2}$



Problem 6:

Positronium is a quasi-atomic system consisting of an electron and a positron. In terms of the ionization energy E_0 of the hydrogen atom, what must be the energy of a photon making a transition from the first excited state of positronium to the ground state?

(A) $\frac{3}{2}E_0$

(B) $\frac{3}{4}E_0$

(C) $\frac{1}{2}E_0$

(D) $\frac{3}{8}E_0$

(E) $\frac{1}{8}E_0$

Problem 7:

The ground states of the helium, neon, and argon atoms are all

(A) $^1S_0$

(B) $^2S_{\frac{1}{2}}$

(C) $^3S_1$

(D) $^1P_1$

(E) $^2P_{\frac{1}{2}}$

Problem 8:

A particle of mass M is in an infinite square well potential

$V = \infty$ for $x < -a,\ a < x$ $\qquad V = 0$ for $-a \leqslant x \leqslant a$

What are the energy eigenfunctions for the two lowest possible energies?

(A) $\frac{1}{\sqrt{a}}\sin\frac{\pi x}{2a}, \frac{1}{\sqrt{a}}\cos\frac{\pi x}{2a}$

(B) $\frac{1}{\sqrt{a}}\sin\frac{\pi x}{a}, \frac{1}{\sqrt{a}}\cos\frac{2\pi x}{a}$

(C) $\frac{1}{\sqrt{a}}\cos\frac{\pi x}{2a}, \frac{1}{\sqrt{a}}\sin\frac{\pi x}{a}$

(D) $\frac{1}{\sqrt{a}}\cos\frac{\pi x}{a}, \frac{1}{\sqrt{a}}\sin\frac{2\pi x}{a}$

(E) $\frac{1}{\sqrt{a}}\cos\frac{2\pi x}{a}, \frac{1}{\sqrt{a}}\sin\frac{4\pi x}{a}$

Problem 9:

The wave function of a particle is $e^{i(kx-\omega t)}$, where x is position, t is time, and k and ω are positive real numbers. The wave function represents a simultaneous eigenstate of

(A) position and momentum

(B) energy and time

(C) energy and momentum

(D) position and time

(E) energy, momentum, position, and time

Problem 10:

Under an exchange of both coordinates and spins, the complete wave function for a system of two electrons must be

(A) antisymmetric

(B) symmetric

(C) additive

(D) incoherent

(E) orthogonal to either independent wave function

Problem 11:

In the Compton effect, a photon with energy E scatters through a 90° angle from a stationary electron of mass m. The energy of the scattered photon is

(A) E

(B) $\frac{E}{2}$

(C) $\frac{E^2}{mc^2}$

(D) $\frac{E^2}{E + mc^2}$

(E) $\frac{E \cdot mc^2}{E + mc^2}$

Problem 12:

A particle of charge e and mass m is trapped in a one-dimentional square well of width 2a with impenetrable walls. The walls are located at $x = -a$ and $x = a$.
If the quantum mechanical states are labled by n ($n = 1$ is the ground state), what is the expectation value of x^2 for very large n ?
(A) 0 (B) $a^2/6$ (C) $a^2/4$ (D) $a^2/3$ (E) $a^2/2$

Problem 13:

Refer to the previous problem.

A small electric field of strength E_0 is applied in the x direction. What is the change of the energy of the first excited state of this particle due to this electric field? (Assume that the total potential, due to the walls and the electric field, is 0 at x=0.)

(A) $\frac{-eE_0a}{2}$

(B) $\frac{-eE_0a}{3}$

(C) 0

(D) $\frac{eE_0a}{3}$

(E) $\frac{eE_0a}{2}$

Problem 14:

If an electron is in a 4f state, the magnitude of its orbital angular momentum is

(A) $\sqrt{2}\hbar$

(B) $\sqrt{3}\hbar$

(C) $\sqrt{6}\hbar$

(D) $2\sqrt{3}\hbar$

(E) $4\sqrt{5}\hbar$

Problem 15:

The Pauli exclusion principle results from the quantum mechanical fact that no two electrons in an atom can

(A) have the same set of quantum numbers
(B) have the same spatial wave function
(C) have the same spin
(D) interact with each other
(E) be in an excited state simultaneously