Advanced E&M, solutions

Problem 1:
(C) RLC resonance circuits
Three impedances in series: $Z = R + i\omega L + 1/(i\omega C)$
$|V| = |I|\,|Z|$
Minimum $|Z|$ gives maximum I. (resonance)
$|Z| = [R^2 + (\omega L - 1/(\omega C))^2]^{1/2}$
$Z_{min} = R$ when $\omega L - 1/(\omega C) = 0$, $\omega^2 = 1/(LC)$
$C = 1/(L\omega^2)$, $L\omega^2 = 2*10^{-6}(2*\pi*1.037*10^8)^2 = 8.5*10^{11}$

Problem 2:
(A) Faraday's law
The magnetic field strength changes from B to zero in some short time interval Δt.
The flux through a circle of radius d/2 changes from $B\pi R^2$ to zero in the time interval Δt.
Faraday's law: $E2\pi d/2 = B\pi R^2/\Delta t$, $E = BR^2/(d\,\Delta t)$.
The force on each charge is $F = qE$. The impulse for each charge is $\Delta p = qE\Delta t$.
The total angular impulse is $\Delta L = 2\,\Delta p\, d/2 = qE\Delta t d = qBR^2$.

Problem 3:
(C) Relativistic E&M
Let K be the frame of the observer at rest on the surface and K' the frame of the observer moving relative to the surface.
$\sigma = Q/A = Q/(LW)$ in K, $\sigma' = Q/A' = Q/(L'W)$ in K'.
$L' = L/\gamma$ (length contraction) so $\sigma' = \gamma\sigma'$.
Therefore $E' = \gamma\sigma/(2\varepsilon_0) = [\sigma/(2\varepsilon_0)]*(1-v^2/c^2)^{-1/2}$.

Problem 4:
(B) Relativistic E&M

Problem 5:
(C) Kirchhoff's loop rule
120 V – 100 V – 10 A*1 Ω – 10 A*R = 0, R = 1Ω.

Problem 6:
(B) Gauss' law:
$4\pi r^2 E(r) = Q_{inside}/\varepsilon_0$.
$4\pi(R^2/4)E(R/2) = (4\pi/\varepsilon_0)\int_0^{R/2} Ar^4 dr$.
$E(R/2) = A(R/2)^3/(5\varepsilon_0))$

Problem 7:
(D) The number of electrons passing any point in the wire per unit time is $\rho * v_{drift} * A = 10^{28} * v_{drift} * \pi * (0.01)^2/s$. The current is $1.6*10^{-19}*10^{28}*v_{drift}*\pi*(0.01)^2$ C/s = 100 A.
$v_{drift} \sim 2*10^{-4}$ m/s.

Problem 8:
(A) Faraday's law
$\varepsilon = -\partial \text{flux}/\partial t = -BdA/dt$. Here dA/dt is equal to a constant C for half a cycle, and -C for the other half of the cycle.

Problem 9:
(B) The electric potential
The distance of P to any point on the ring is $r = (R^2 + x^2)^{1/2}$. $V = kQ/r$.

Problem 10:
(A) The electric force,
The force on the particle is $kQq/(R^2 + x^2) * x/(R^2 + x^2)^{1/2} \sim kQqx/R^3$ towards the origin.
It is a restoring force. $F = -\alpha x$, $\omega^2 = \alpha/m$.

Problem 11:
(B) Faraday's law
emf $= -(\partial B/\partial t)A = -(150T/s)0.01m^2 = -1.5V$ (Lenz's rule)
$5V - 1.5V = I*10\Omega$, $I = 0.35A$

Problem 12:
(A) $P = IV = V^2/R$. Here V is constant.

Problem 13:
(C) You can find E from Gauss' law.

Problem 14:
(D) Method of images
Method of images yields **E**, $\sigma = \varepsilon_0 E$ from Gauss' law..

Problem 15:
(C) State this differently, in words, not using an equation.