

## Waves and optics, solutions

### Problem 1:

(D) Keplerian telescope: separation:  $f_1 + f_2$ , magnification:  $f_1/f_2$ .

### Problem 2:

(B) Polarizers: transmitted intensity/incident intensity =  $0.5 \cdot \cos^2(\theta) \cdot \cos^2(90^\circ - \theta)$ .  
Maximum at  $\theta = 45^\circ$ .

### Problem 3:

(B) Holograms record an interference pattern. A reference beam interferes with a beam transmitted through or reflected by the object. Interference patterns contain amplitude and phase information.

### Problem 4:

(E) Thin film-interference: When a light wave reflects from a medium with a larger index of refraction, then the phase shift of the reflected wave with respect to the incident wave is  $\pi$  ( $180^\circ$ ). When a light wave reflects from a medium with a smaller index of refraction, then the phase shift of the reflected wave with respect to the incident wave is zero.

### Problem 5:

(A) Diffraction limited optics: Light rays from a distant source arrive nearly parallel. Neglecting diffraction, the image of a point will be a disk of approximate diameter  $d$ . We want the diameter of the Airy disk (bright center of the diffraction pattern) to be approximately  $d$ . The first minimum of the diffraction pattern occurs at approximately  $d \sin \theta = \lambda$ . We then have  $\sin \theta \sim d/D$ . Therefore  $d \sim (\lambda D)^{1/2}$ .

### Problem 6:

(E) Phase and group velocity: phase velocity:  $\omega/k$ , group velocity:  $d\omega/dk$ .  
Information (energy) moves with a speed equal to the group velocity.

### Problem 7:

(E) Polarization: For plane polarization the y- and z-components must be in phase or  $180^\circ$  out of phase.

### Problem 8:

(D) Diffraction and interference patterns:  
Single slit minimum falls on double slit maximum.  
Single slit minimum:  $w \sin \theta = m\lambda$ .  
Double slit maxima:  $d \sin \theta = n\lambda$ .  
 $w/d = m/n$ ,  $nd = mw$ . Since  $d > w$ , the only possible answer is D.

### Problem 9:

(C) Mirrors and thin lenses:  
mirror equation:  $1/x_o + 1/x_i = 1/f$ , negative image distance  $\rightarrow$  virtual image.

### Problem 10:

(E) Diffraction and interference patterns:

**Problem 11:**

(B) Double slit interference:  $d \sin\theta = \lambda$ ,  $d = 0.01/2000$  m.

Small angles:  $\theta \sim \lambda/d = 0.1$  rad.  $2\pi$  rad = 360 deg, 1 rad  $\sim$  60 deg, 0.1 rad  $\sim$  6 deg.

**Problem 12:**

(D) The speed of light in a dielectric:  $c^2 = 1/(\epsilon_0\mu_0)$ ,  $v^2 = 1/(\epsilon\mu)$ ,  $v^2 = c^2/(\kappa_e\kappa_m)$

**Problem 13:**

(E) Traveling waves:

$y = A\sin(\omega t - kx)$  represents a wave traveling into the positive x-direction with speed

$$v = \omega/k = \lambda/T$$

**Problem 14:**

(A) Thin film-interference: The reflections from the front and rear surface of the layer both produce a phase shift of  $\pi$ . Traversing the layer twice (back and forth) also must introduce a phase shift of  $\pi$ . The waves reflected from the front and back surface of the layer must be out of phase by  $1/2$  wavelength to destructively interfere.

**Problem 15:**

(C) Total internal reflection:  $\sin\theta_c = n_{\text{small}}/n_{\text{big}} = 1/1.33 = 3/4$ ,  $\theta_c \sim 50^\circ$ .