

Solutions

Common themes:

Theme 1: Coulomb's law, the electric field of a point charge.

Theme 2: The electromagnetic force $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.

Theme 3: Gauss' law and Ampere's law alone can be used to find the electric and magnetic field in situations with enough symmetry (spherical, cylindrical planar).

Theme 4: Capacitors in series and parallel, energy stored in a capacitor $U = Q^2/C = CV^2$.

Theme 5: The principle of superposition.

Theme 6: Faraday's law

Problem 1:

(D) $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0$, B is proportional to I. (theme 2)

Problem 2:

(C) Both the electric and the magnetic force drop off as $1/r$. (theme 3)

Problem 3:

$$(C) \quad W = \int_0^Q \frac{1}{4\pi\epsilon_0} \frac{q dq}{R} = \frac{1}{8\pi\epsilon_0} \frac{q^2}{R} \Big|_0^Q = \frac{1}{8\pi\epsilon_0} \frac{Q^2}{R}.$$

or: $C = Q/V$, $= Q/(kQ/r) = r/k$, $U = \frac{1}{2} Q^2/C$. (theme 4)

Problem 4:

(D)

Problem 5:

(D) $F = kQ_1Q_2/r^2$ is the magnitude of the initial force. Initially $Q_1 = Q_2 = Q$. After the touching sequence $Q_1 = Q/2$, $Q_2 = 3Q/4$, $F = 3F/8$. (theme 1)

Problem 6:

(E)

Problem 7:

(E) The net electrostatic force on the charge Q at the center is zero since the force on Q due to each negative charge is balanced by the force due to the negative charge at the diagonally opposite corner of the cube. (theme 5)

Problem 8:

(C) Motional emf:

The section of the wire extending from the bottom center of the cylinder radially outward to radius R rotates with angular frequency $\omega = N2\pi$ rad/sec in the magnetic field. The force on an electron in this section is has magnitude $qvB = q\omega rB$. The magnitude of the potential difference is $(1/q) \int_0^R F dr = \pi NBR^2$.

or

$d\Phi_B/dt$ (filamentary circuit with moving parts, constant B) = motional emf.

Problem 9:

(E) The particle starts from rest. It accelerates in a direction parallel or anti-parallel to \mathbf{E} and always moves parallel or anti-parallel to \mathbf{B} . (theme 2)

Problem 10:

(A) $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. (theme 2)

Problem 11:

(C) $Q_1 = C_1V$, $Q_2 = C_2V$, $V = 5V$.

$Q_f = Q_2 - Q_1 = (C_2 - C_1)V$. $V_f = Q_f/C_f = (C_2 - C_1)V/(C_2 + C_1) = 5/3$. (theme 4)

Problem 12:

(A) Ampere's law and the principle of superposition

Right hand rule: $\mathbf{B} = B\mathbf{j}$. Each conductor contributes $B/2$.

$2\pi(d/2)B/2 = \mu_0 I_{\text{through}\Gamma} = \mu_0 J\pi(d/2)^2$. (themes 3 and 5)

Problem 13:

(C) Flux = $B\pi R^2 \cos\omega t$

emf = $\omega B\pi R^2 \sin\omega t = E_0 \sin\omega t$

$\omega = E_0/(B\pi R^2)$. (theme 6)

Problem 14:

(C) Self inductance L : $|\mathcal{E}| = L|dI/dt| = |dF/dt|$

$F = NBA$, $B = \mu_0(N/L)I$, $L \propto N^2$.

Problem 15:

(E) $Q/C + R dQ/dt = 0$ $dQ/dt = -Q/(RC)$, $Q = Q_0 \exp(-t/RC)$,

$E = Q^2/C = E_0 \exp(-2t/RC)$, $1/2 = \exp(-2t_{1/2}/RC)$, $t_{1/2} = \ln(2)RC/2$. (theme 4)