Solutions

Common themes:
Theme 1: relationships between potential energy, kinetic energy and speed
Theme 2: Kepler's laws

Problem 1:
(A) This is an inelastic collision. Momentum is conserved, but kinetic energy changes.
$v_i \propto \sqrt{h_0}$, $v_f = v_i/4$, $v_f = \propto\sqrt{h}$. (theme 1)

Problem 2:
(B) $\mathbf{a} = \mathbf{g} - a\mathbf{v}$. Only at the top is $\mathbf{v} = 0$.

Problem 3:
(D) The component of **g** tangent to the curve is $g(dy/ds) = gdy/(dy^2 + dx^2)^{1/2}$
$= g(xdx/2)/((x^2/4 + 1)dx^2)^{1/2} = g(x/2)/(x^2/4 + 1)^{1/2} = gx/(x^2 + 4)^{1/2}$.

Problem 4:
(A) What are the possible orbits in an attractive 1/r potential? (theme 2)

Problem 5:
(E) $x_{CM} = (\int_0^L \rho x dx)/(\int_0^L \rho dx) = (\int_0^L x^2 dx)/(\int_0^L x dx) = 2L/3$
or: What is the only possible answer given that the density increases with x?

Problem 6:
(C) Angular momentum of interacting objects is conserved, but angular position is not.

Problem 7:
(E) Momentum conservation:

$$mv = (m + M)V$$

Work-kinetic energy theorem: (theme 1)

$$\frac{1}{2}(m + M)V^2 = \mu_k (m + M)gs, \quad s = \frac{V^2}{2\mu_k g} = (\frac{v^2}{2\mu_k g})(\frac{m}{m + M})^2$$

Problem 8:
(E) $bx_{max}^2/2 = \frac{1}{2} mv^2$, $x_{max} = 1$. (theme 1)

Problem 9:
(E) Newton's law of gravitation, centripetal acceleration
We need $GMm/r^2 > m\omega^2 r$.

$$\frac{GMm}{r^2} = m\omega^2 r, \quad \omega = \sqrt{\frac{GM}{r^3}} = \sqrt{\frac{4\pi G\rho}{3}}, \quad T = \frac{2\pi}{\omega} = \left(\frac{3\pi}{\rho G}\right)^{\frac{1}{2}}$$

Problem 10:
(C) Gravitational potential energy

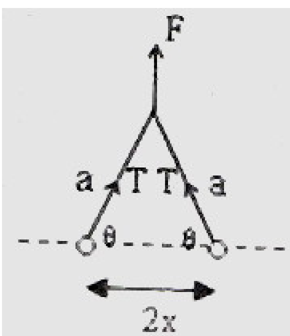
Problem 11:

(D) $\Delta T = \frac{1}{2} I\left(\omega_0^2 - \omega'^2\right) = \frac{1}{2} \times 4 \times \left(80^2 - 40^2\right) = 9600\text{J}$ 。

Problem 12:
(C) $\Delta P = \int F dt$ = area under curve. $\Delta P = 12$ Ns.

Problem 13:
(A) Kepler's third law: $(T_1/T_2)^2 = (R_1/R_2)^3 = (1.01)^3 = (1 + 0.01)^3 \sim 1 + 0.03$
$(T_1/T_2)^2 = (1 + x)^2 \sim 1 + 2x$. $x = 0.015$. (theme 2)

Problem 14:
(B) With reference to the figure, $F = 2T\sin\theta$ so that $T = F/(2\sin\theta)$ where 'T' is the tension in the string.



The vertical force on each particle is $T\sin\theta$, $2T\sin\theta = F$.
The horizontal force on each particle is $T\cos\theta = F\cos\theta/(2\sin\theta) = F/(2\tan\theta)$.
Therefore, the magnitude of acceleration of each particle is given by
$dv_x/dt = F/(2m \tan\theta) = F/[2m\sqrt{(a^2 - x^2)}/x] = Fx/(2m\sqrt{(a^2 - x^2)})$.

Problem 15:
(D) The coefficient of restitution
Let E_i be the energy after the ith bounce. $E_1/E_0 = E_{i+1}/E_i = H/H_0$.
$v_{i+1}/v_i = \text{sqrt}(H/H_0) = (0.8)^{1/2}$, $v_{n-1} = v_0(0.8)^{(N-1)/2}$.
$v_0 = (2gH_0)^{1/2}$. (theme 1)