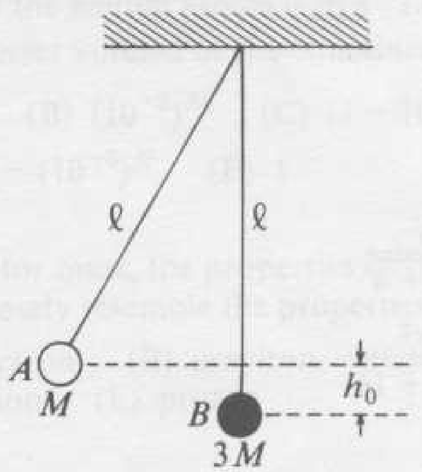
Mechanics

Problem 1:



Two small spheres of putty, A and B, of mass M and $3M$, respectively, hang from the ceiling on strings of equal length ℓ. Sphere A is drawn aside so that it is raised to a height h_0 as shown above and then released. Sphere A collides with sphere B; they stick together and swing to a maximum height h equal to

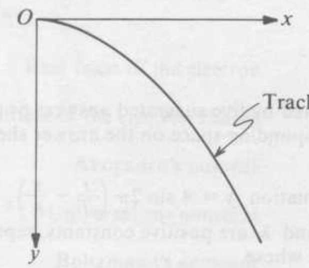
(A) $\frac{1}{16}h_0$ (B) $\frac{1}{8}h_0$ (C) $\frac{1}{4}h_0$
(D) $\frac{1}{3}h_0$ (E) $\frac{1}{2}h_0$

Problem 2:

A rock is thrown vertically upward with initial speed v_0. Assume a friction force proportional to $-\mathbf{v}$, where $\mathbf{v}$ is the velocity of the rock, and neglect the buoyant force exerted by air. Which of the following is correct?

(A) The acceleration of the rock is always equal to $\mathbf{g}$.
(B) The acceleration of the rock is equal to $\mathbf{g}$ only at the top of the flight.
(C) The acceleration of the rock is always less than $\mathbf{g}$.
(D) The speed of the rock upon return to its starting point is v_0.
(E) The rock can attain a terminal speed greater than v_0 before it returns to its starting point.

Problem 3:



A particle is initially at rest at the top of a curved frictionless track. The x- and y-coordinates of the track are related in dimensionless units by $y = \frac{x^2}{4}$, where the positive y-axis is in the vertical downward direction. As the particle slides down the track, what is its tangential acceleration?

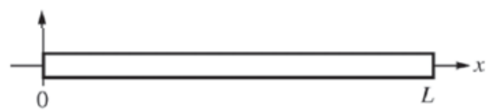
(A) 0
(B) g
(C) $\frac{gx}{2}$
(D) $\frac{gx}{\sqrt{x^2+4}}$
(E) $\frac{gx^2}{\sqrt{x^2+16}}$

Problem 4:

A satellite orbits the Earth in a circular orbit. An astronaut on board perturbs the orbit slightly by briefly firing a control jet aimed toward the Earth's center. Afterward, which of the following is true of the satellite's path?

(A) It is an ellipse.
(B) It is a hyperbola.
(C) It is a circle with larger radius.
(D) It is a spiral with increasing radius.
(E) It exhibits many radial oscillations per revolution.

Problem 5:

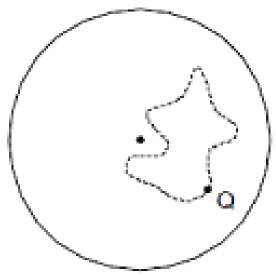


A rod of length L and mass M is placed along the x-axis with one end at the origin, as shown in the figure. The rod has linear mass density $\lambda = \frac{2M}{L^2}x$, where x is the distance from the origin. Which of the following gives the x-coordinate of the rod's center of mass?

(A) $\frac{1}{12}L$ (B) $\frac{1}{4}L$

(C) $\frac{1}{3}L$ (D) $\frac{1}{2}L$ (E) $\frac{2}{3}L$

Problem 6:



The diagram above show a top view of a phonograph turntable mounted on a frictionless bearing. Initially, the turntable is at rest and a massive bug is asleep at point Q. The bug wakes up, takes a walk such as the one indicated by the dotted line, returns to Q, and goes back to sleep. Afterward, the turntable is

(A) again at rest in the same position

(B) at rest, but in the same position only if the walk did not encircle the pivot point

(C) at rest, but not necessarily in the same position whether or not the walk encircled the pivot point

(D) at rest only if the walk did not encircle the pivot point

(E) not necessarily at rest, whether or not the pivot point was encircled

Problem 7:

A bullet of mass m traveling at speed v strikes a block of mass M, initially at rest, and is embedded in it as shown above. How far will the block with the bullet embedded in it slide on a rough horizontal surface of coefficient of kinetic friction μ_k before it comes to rest?

(A) $(\frac{m+M}{m})(\frac{v^2}{2\mu_k g})$

(B) $(\frac{m+M}{M})(\frac{v^2}{2\mu_k g})$

(C) $(\frac{m+M}{M})^2(\frac{v^2}{2\mu_k g})$

(D) $(\frac{m}{m+M})(\frac{v^2}{2\mu_k g})$

(E) $(\frac{m}{m+M})^2(\frac{v^2}{2\mu_k g})$

Problem 8:

A particle of mass M moves along the x-axis under the influence of a conservative field with the potential energy $V = \frac{b}{2}x^2$. If the particle starts from rest at x = 1, its maximum velocity is

(A) $\sqrt{\frac{Mb}{2}}$

(B) $\sqrt{Mb}$

(C) $\sqrt{2Mb}$

(D) $\sqrt{\frac{b}{2M}}$

(E) $\sqrt{\frac{b}{M}}$

Problem 9:

A spherical neutron star has a uniform mass density ρ. What is the period of rotation below which material will fly off the equator? (Use nonrelativistic mechanics and let G be the universal gravitational constant.)

(A) $\frac{3}{4\pi G}$

(B) $\frac{4}{3\pi G}$

(C) $\left(\frac{3}{8\pi\rho G}\right)^{\frac{1}{2}}$

(D) $\left(\frac{\pi}{\rho G}\right)^{\frac{1}{2}}$

(E) $\left(\frac{3\pi}{\rho G}\right)^{\frac{1}{2}}$

Problem 10:

A planet of mass m moves about the Sun of mass M. G is Newton's constant, r is the planet's distance from the Sun, and v is the planet's speed. Except for an additive constant the planet's potential energy is

(A) $\frac{1}{2}mv^2 + \frac{GMm}{r}$

(B) $\frac{1}{2}mv^2 - \frac{GMm}{r}$

(C) $-\frac{GMm}{r}$

(D) $-\frac{GMm}{r^2}$

(E) mgr

Problem 11:

A cylinder with moment of inertia 4 $kg{\cdot}m^2$ about a fixed axis initially rotates at 80 radians per second about this axis. A constant torque as applied to slow it down to 40 radius per second.

The kinetic energy lost by the cylinder is

(A) 80 J

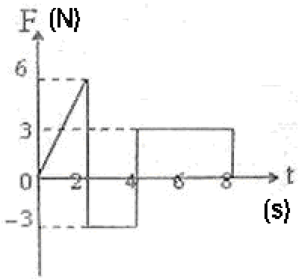
(B) 800 J

(C) 4000 J

(D) 9600 J

(E) 19,200 J

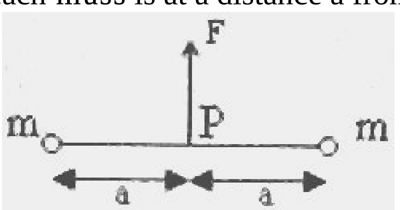
Problem 12:
The force F acting on a particle of mass m is indicated by the force-time graph shown below.



The change in momentum of the particle over the time interval from zero to 8 s is
(A) 24 Ns (B) 20 Ns
(C) 12 Ns (D) 6 Ns
(E) zero

Problem 13:
A satellite is launched in a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius 1.01 R. The period of the second satellite is longer than that of the first one (approximately) by
(A) 1.5%
(B) 0.5%
(C) 3%
(D) 1%
(E) 2%

Problem 14:
Two particles of mass m each are tied at the ends of a light string of length 2a. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance a from the center P (as shown in the figure).



Now, the midpoint of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes 2x is
(A) $Fa/2m\sqrt{(a^2-x^2)}$
(B) $Fx/2m\sqrt{(a^2-x^2)}$
(C) $Fx/2ma$
(D) $F\sqrt{(a^2-x^2)}/2mx$
(E) $F/2m$

Problem 15:

A ball rises after one bounce to 80 percent of the height from which it is released from rest. If air resistance is negligible and the ball is released from rest from a height H_0, what is its speed just before it hits the floor for its Nth bounce?

(A) $(0.8)^{2N}\sqrt{2gH_0}$
(B) $(0.8)^{N}\sqrt{2gH_0}$
(C) $(0.8)^{N-\frac{1}{2}}\sqrt{2gH_0}$
(D) $(0.8)^{\left(\frac{N-1}{2}\right)}\sqrt{2gH_0}$
(E) $(0.8)^{N-1}\sqrt{2gH_0}$