Quantum Mechanics 1, solutions

Problem 1:
(B) Ground state of the 1D harmonic oscillator
This wave function is the ground-state wave function of the harmonic oscillator.

Problem 2:
(A) Symmetry!
The average value of the potential energy is unchanged.

Problem 3:
(E) Postulates of Quantum Mechanics
Any $\Psi(\mathbf{r},t_0)$ can be expanded in terms of eigenfunctions,
$\Psi(\mathbf{r},t_0) = \sum_a c_a \Psi_a(\mathbf{r})$.
The probability that a measurement at $t = t_0$ will yield the eigenvalue a' is
$P_{a'} = |c_{a'}|^2/\sum_a |c_a|^2$.

Problem 4:
(A) The Pauli exclusion principle
Postulate: Every elementary particle is either a **fermion or a boson**. A state of many identical particles is totally **antisymmetric** with respect to the interchange of any two particles if they are fermions, and it is totally **symmetric** if they are bosons. No two identical fermions can have exactly the same set of quantum numbers. This is called the Pauli exclusion principle.

Problem 5:
(D) Force on a magnetic dipole
The magnetic dipole associated with the spin only experiences a force in an inhomogeneous magnetic field.

Problem 6:
(C) Energy of a magnetic dipole in a magnetic field
The magnetic dipole moment $\boldsymbol{\mu}$ of an atom is proportional to its angular momentum $\mathbf{J}$. The energy of a magnetic dipole in an external magnetic field is $E = -\boldsymbol{\mu}\cdot\mathbf{B}$. If B, for example, points into the z-direction, then $E = -\mu_z B$. Since μ_z is proportional to J_z, measurement of E will yield values proportional to m. For a given j, there are 2j + 1 possible values of m, from –j to +j in integer steps.

Problem 7:
(D) Population inversion
Transition c is the laser transition.
3-level optical pumping schemes are also possible, but in a 3-level scheme the upper laser level has to be long-lived or metastable.

Problem 8:
(C) Tunneling
This is a scattering problem, not a bound-state problem. We have tunneling.

Problem 9:
(D) Normalization of a wave function
$\int_{\text{all space}}|\Psi(\mathbf{r},t)|^2 d^3r = 1$, here we have a 1-dimensional problem.
$\int_0^{2\pi}|A|^2 d\varphi = 1$.

Problem 10:
(A) The photoelectric effect
Experimental setup

Problem 11:
(D) The photoelectric effect
The energy carried by an EM wave is quantized.

Problem 12:
(D) The photoelectric effect
W is the work function of the metal.

Problem 13:
(D) Compton scattering
The quantity $h/(m_pc)$ is called the Compton wavelength for photon-proton scattering. The Compton wavelength is the amount by which the photon's wavelength changes when it scatters through 90°.

Problem 14:
(E) Identical particles
The total wave function is symmetric under exchange. The particles must be bosons.

Problem 15:
(D) Electric dipole transitions
In dipole transitions the spin is not involved.