Quantum Mechanics

Problem 1:

The wave function $\psi(x) = A \exp\left\{-\frac{b^2x^2}{2}\right\}$, where A and b are real constants, is a normalized eigenfunction of the Schrödinger equation for a particle of mass M and energy E in a one dimensional potential $V(x)$ such that $V(x) = 0$ at $x = 0$. Which of the following is correct?

(A) $V = \frac{\hbar^2 b^4}{2M}$

(B) $V = \frac{\hbar^2 b^4 x^2}{2M}$

(C) $V = \frac{\hbar^2 b^6 x^4}{2M}$

(D) $E = \hbar^2 b^2 (1 - b^2 x^2)$

(E) $E = \frac{\hbar^2 b^4}{2M}$

Problem 2:

In perturbation theory, what is the first order correction to the energy of a hydrogen atom (Bohr radius a_0) in its ground state due to the presence of a static electric field E?

(A) Zero

(B) eEa_0

(C) $3eEa_0$

(D) $\frac{8e^2Ea_0^3}{3}$

(E) $\frac{8e^2E^2a_0^3}{3}$

Problem 3:

A system is known to be in the normalized state described by the wave function

$$\psi(\theta, \varphi) = \frac{1}{\sqrt{30}}(5\,Y_4^3 + Y_6^3 - 2\,Y_6^0),$$

where the $Y_\ell^m(\theta, \varphi)$ are the spherical harmonics. The probability of finding the system in a state with azimuthal orbital quantum number $m = 3$ is

(A) 0, (B) 1/15, (C) 1/6,
(D) 1/3, (E) 13/15

Problem 4:

The wave function for identical fermions is anti-symmetric under particle interchange. Which of the following is a consequence of this property?

(A) Pauli exclusion principle
(B) Bohr correspondence principle
(C) Heisenberg uncertainty principle
(D) Bose-Einstein condensation
(E) Fermi's golden rule

Problem 5:

The hypothesis that an electron possesses spin is qualitatively significant for the explanation of all of the following topics EXCEPT the

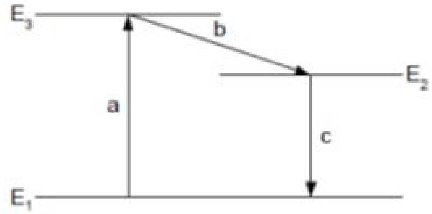
(A) structure of the periodic table
(B) specific heat of metals
(C) anomalous Zeeman effect
(D) deflection of a moving electron by a uniform magnetic field
(E) fine structure of atomic spectra

Problem 6:

If the hyperfine structure is ignored, an atomic state $^2P_{3/2}$ in a weak external field will be split into a number of states equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Problem 7:



The diagram above shows the levels and transitions of a ruby laser. All of the following statements about the laser are correct EXCEPT:

(A) The laser works because of population inversion.

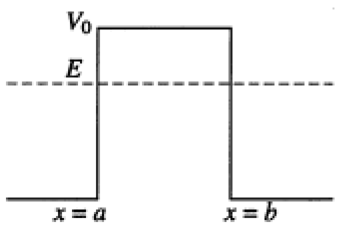
(B) E_2 is the energy of a metastable state.

(C) Transition a involves absorption of radiation.

(D) Transition b involves stimulated emission.

(E) The laser must be optically pumped by radiation of frequency $f = \dfrac{E_3 - E_1}{h}$.

Problem 8:

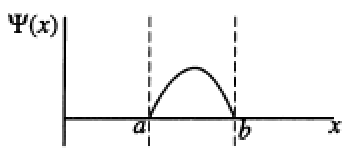


Consider a potential of the form

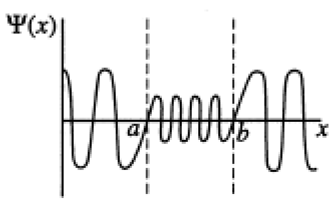
$$V(x) = 0,\ x \le a$$
$$V(x) = V_0,\ a < x < b$$
$$V(x) = 0,\ x \ge b$$

as shown in the figure above. Which of the following wave functions is possible for a particle incident from the left with energy $E < V_0$?

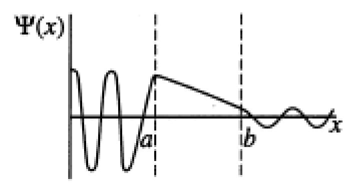
(A)



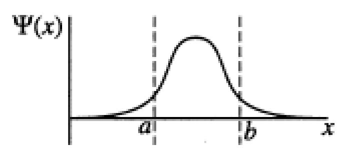
(B)



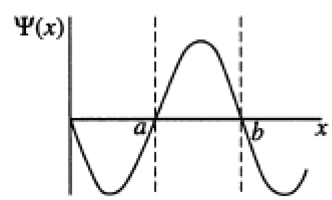
(C)



(D)



(E)



Problem 9:

Eigenfunctions for a rigid dumbbell rotating about its center have a ϕ dependence of the form $\psi(\phi) = Ae^{im\phi}$, where m is a quantum number and A is a constant. Which of the following values of A will properly normalize the eigenfunction?

(A) $\sqrt{2\pi}$

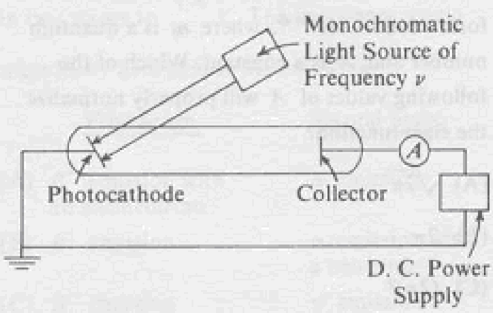
(B) 2π

(C) $(2\pi)^2$

(D) $\frac{1}{\sqrt{2\pi}}$

(E) $\frac{1}{2\pi}$

Problem 10:



In this apparatus, the photocathode and the collector are made from the same material. The potential V of the collector, measured relative to ground, is initially zero and is then increased or decreased monotonically. The effect is described by Einstein's photoelectric equation

$$|eV| = h\nu - W.$$

When the photoelectric equation is satisfied and applicable to this situation, V is the

(A) negative value at which the current stops
(B) negative value at which the current starts
(C) positive value at which the current stops
(D) positive value at which the current starts
(E) voltage induced when the light is on

Problem 11:

The photoelectric equation is derived under the assumption that

(A) electrons are restricted to orbits of angular momentum $n\hbar$, where n is an integer
(B) electrons are associated with waves of wavelength $\lambda = h/p$, where p is momentum
(C) light is emitted only when electrons jump between orbits
(D) light is absorbed in quanta of energy $E = h\nu$
(E) light behaves like a wave

Problem 12:

The quantity W in the photoelectric equation is the

(A) energy difference between the two lowest electron orbits in the atoms of the photocathode
(B) total light energy absorbed by the photocathode during the measurement
(C) minimum energy a photon must have in order to be absorbed by the photocathode
(D) minimum energy required to free an electron from its binding to the cathode material
(E) average energy of all electrons in the photocathode

Problem 13:

Photons of wavelength λ scatter elastically on free protons initially at rest. The wavelength of the photons scattered at 90° is increased by

(A) $\lambda/137$
(B) $\lambda/1836$
(C) h/m_ec, where h is Planck's constant, m_e the rest mass of an electron, and c the speed of light
(D) h/m_pc, where h is Planck's constant, m_p the rest mass of a proton, and c the speed of light
(E) zero

Problem 14:

A system containing two identical particles is described by a wave function of the form

$$\psi = \frac{1}{\sqrt{2}}\left[\psi_\alpha(x_1)\,\psi_\beta(x_2) + \psi_\beta(x_1)\,\psi_\alpha(x_2)\right]$$

where x_1 and x_2 represent the spatial coordinates of the particles and α and β represent all the quantum numbers, including spin, of the states that they occupy. The particles might be

(A) electrons
(B) positrons
(C) protons
(D) neutrons
(E) deuterons

Problem 15:

Which of the following is NOT compatible with the selection rule that controls electric dipole emission of photons by excited states of atoms?

(A) Δn may have any negative integral value.
(B) $\Delta \ell = \pm 1$
(C) $\Delta m_\ell = 0, \pm 1$
(D) $\Delta s = \pm 1$
(E) $\Delta j = \pm 1$