

Solutions

Problem 1:

(C) Length contraction

$$L = L_0/\gamma, \gamma = 5/3, v = 0.8c.$$

Problem 2:

(A) Length contraction

$$L = 4m/\gamma = 4m \cdot (3/5) = 2.4 \text{ m.}$$

Problem 3:

(E) The simultaneity problem

Problem 4:

(A) Particles with zero rest mass

Problem 5:

(D) Relativistic energy and momentum

$$pc = \gamma mvc = 5\text{MeV}$$

$$E = \gamma mc^2 = 10\text{MeV}$$

$$pc/E = v/c = 1/2$$

Problem 6:

(A) Definition of acceleration

$$a = dv/dt = (dv/dx)(dx/dt) = (dv/dx)v$$

Problem 7:

(D) The Lagrangian $L = T - U$:

$$T = 1/2M(dX/dt)^2 + 1/2m(d(X + x)/dt)^2, U = 1/2kx^2.$$

Problem 8:

(E) Centripetal acceleration

$$T\cos(\theta/2) = mg, T\sin(\theta/2) = mv^2/r = m\omega^2r. T^2 = m^2(g^2 + \omega^4r^2).$$

Problem 9:

(D) Time dilation

$$t = \gamma\tau = 10^{-8}\text{s}/(5/9)^{1/2} = 3 \cdot 10^{-8}\text{s}/(5)^{1/2}. d = vt = 6\text{m}/(5)^{1/2}$$

Problem 10:

(E) Relativistic expression for energy, $E = \gamma mc^2$

$$\gamma 494 = 938, \gamma = 938/494, 1 - v^2/c^2 = 0.277, v = 0.85c$$

Problem 11:(C) **The space-time interval** $d = (c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2)^{1/2}$ is invariant under the Lorentz transformation. Therefore $|d| = |c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2|^{1/2}$ is invariant under the Lorentz transformation.Here $|d| = |4 - 4 - 4|^{1/2} = 2$.**Problem 12:**(A) **Damped harmonic oscillator**

$$m d^2 x / dt^2 = -kx - b dx / dt.$$

Solution: $x(t) = A \exp(-\beta t) \cos(\omega_1 t - \delta)$, $\beta = b / (2m)$, $\omega_1^2 = \omega_0^2 - \beta^2$, $\omega_1^2 < \omega_0^2$, $T_1 > T_0$.**Problem 13**(B) **Conservative forces**

$$F = -dU / dx.$$

Problem 14:(A) **The Hamiltonian** $H = T + V$. H is a function of x and p .**Problem 15:**

(A) The principle of least action

Define the action S as the integral of the Lagrangian between two instances of time.

$$S = \int_{t_1}^{t_2} L(q, dq/dt, t) dt.$$

 $\delta S = 0$, the path taken by the system between times t_1 and t_2 and configurations q_1 and q_2 is the one for which the action is stationary (no change) to first order.