Advanced Mechanics

For problems 1-3:

A car of rest length 5 meters passes through a garage of rest length 4 meters. Due to the relativistic Lorentz contraction, the car is only 3 meters long in the garage's rest frame. There are doors on both ends of the garage, which open automatically when the front of the car reaches them and close automatically when the rear passes them. The opening or closing of each door requires a negligible amount of time.

Problem 1:

The velocity of the car in the garage's rest frame is

(A) 0.4 c
(B) 0.6 c
(C) 0.8 c
(D) greater than c
(E) not determinable from the data given

Problem 2:

The length of the garage in the car's rest frame is

(A) 2.4 m
(B) 4.0 m
(C) 5.0 m
(D) 8.3 m
(E) not determinable from the data given

Problem 3:

Which of the following statements is the best response to the question:

"Was the car ever inside a closed garage?"

(A) No, because the car is longer than the garage in all reference frames.
(B) No, because the Lorentz contraction is not a "real" effect.
(C) Yes, because the car is shorter than the garage in all reference frames.
(D) Yes, because the answer to the question in the garage's rest frame must apply in all reference frames.
(E) There is no unique answer to the question, as the order of door openings and closings depends on the reference frame.

Problem 4:

If a newly discovered particle X moves with a speed equal to the speed of light in vacuum, then which of the following must be true?

(A) The rest mass of X is zero.
(B) The spin of X equals the spin of a photon.
(C) The charge of X is carried on its surface.
(D) X does not spin.
(E) X cannot be detected.

Problem 5:

What is the speed of a particle having a momentum of 5 MeV/c and a total relativistic energy of 10 MeV ?

(A) c

(B) 0.75 c

(C) $\frac{1}{\sqrt{3}}\, c$

(D) $\frac{1}{2}\, c$

(E) $\frac{1}{4}\, c$

Problem 6:

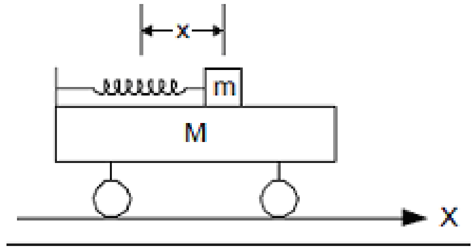
A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to

$$v(x) = \beta x^{-n},$$

where β and n are constants and x is the position of the particle. What is the acceleration of the particle as a function of x ?

(A) $-n\beta^2 x^{-2n-1}$
(B) $-n\beta^2 x^{-n-1}$
(C) $-n\beta^2 x^{-n}$
(D) $-\beta x^{-n+1}$
(E) $-\beta x^{-2n+1}$

Problem 7:



A car of mass M moves horizontally along the X-axis, as shown above. A mass m is attached to the car by a spring having spring constant k. The x-coordinate measures of the position of m; x = 0 corresponds to the position of m in which the spring in neither stretched nor compressed. The axes x and X are parallel.

What is the Lagrangian of this system?

(A) $\frac{1}{2}M\dot{X}^2 + \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$

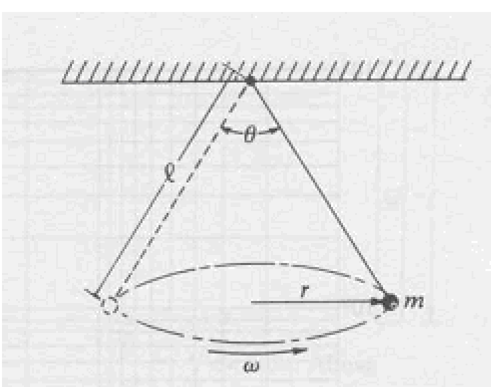
(B) $\frac{1}{2}M\dot{X}^2 + \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$

(C) $\frac{1}{2}M\dot{X}^2 + \frac{1}{2}m(\dot{X}^2 + \dot{x}^2) - \frac{1}{2}kx^2$

(D) $\frac{1}{2}M\dot{X}^2 + \frac{1}{2}m(\dot{X}^2 + 2\dot{x}\dot{X} + \dot{x}^2) - \frac{1}{2}kx^2$

(E) $\frac{1}{2}(M + m)(\dot{X}^2 + \dot{x}^2) - \frac{1}{2}kx^2$

Problem 8:



The figure represents a point mass m attached to the ceiling by a cord of fixed length ℓ. If the point mass moves in a horizontal circle of radius r with uniform angular velocity ω, the tension in the cord is

(A) $mg\left(\frac{r}{\ell}\right)$

(B) $mg\cos\left(\frac{\theta}{2}\right)$

(C) $\dfrac{m\omega r}{\sin\left(\frac{\theta}{2}\right)}$

(D) $m(\omega^2 r^2 + g^2)^{\frac{1}{2}}$

(E) $m(\omega^4 r^2 + g^2)^{\frac{1}{2}}$

Problem 9:

The lifetime of a particular meson at rest is 10^{-8} second and its mass is 10^{-25} gram. If its velocity in the laboratory is $2*10^{8}$ meters per second, how far will it travel in one lifetime, if both distance and lifetime are measured in the laboratory frame?

(A) 10^{-3} m

(B) 2 m

(C) $\sqrt{5}$ m

(D) $\frac{6}{\sqrt{5}}$ m

(E) $\frac{9}{\sqrt{5}}$ m

Problem 10:

A positive kaon (K^+) has a rest mass of 494 MeV/c^2, whereas a proton has a rest mass of 938 MeV/c^2. If a kaon has a total energy that is equal to the proton rest energy, the speed of the kaon is most nearly

(A) 0.25 c
(B) 0.40 c
(C) 0.55 c
(D) 0.70 c
(E) 0.85 c

Problem 11:

Two observers O and O' observe two events, A and B. The observers have a constant relative speed of 0.8 c. In units such that the speed of light is 1, observer O obtained the following coordinates:

Event A: $x = 3,\ y = 3,\ z = 3,\ t = 3$

Event B: $x = 5,\ y = 3,\ z = 1,\ t = 5$

What is the length of the space-time interval between these two events, as measured by O'?

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) 3 (E) $2\sqrt{3}$

Problem 12:

A particle of mass m undergoes harmonic oscillation with period T_0. A force f proportional to the speed v of the particle, $f = -bv$, is introduced. If the particle continues to oscillate, the period with f acting is

(A) larger than T_0
(B) smaller than T_0
(C) independent of b
(D) dependent linearly on b
(E) constantly changing

For problems 13 – 15:

The potential energy of a body constrained to move on a straight line is kx^4 where k is a constant. The position of the body is x, its speed v, its linear momentum p, and its mass m.

Problem 13:

The force on the body is

(A) $\frac{1}{2}mv^2$ (B) $-4kx^3$ (C) kx^4
(D) $-\frac{kx^5}{5}$ (E) mg

Problem 14:

The Hamiltonian function for this system is

(A) $\frac{p^2}{2m} + kx^4$ (B) $\frac{p^2}{2m} - kx^4$ (C) kx^4
(D) $\frac{1}{2}mv^2 - kx^4$ (E) $\frac{1}{2}mv^2$

Problem 15:

The body moves from x_1 at time t_1 to x_2 at time t_2. Which of the following quantities is an extremum for the $x - t$ curve corresponding to this motion, if end points are fixed?

(A) $\int_{t_1}^{t_2} \left(\frac{1}{2}mv^2 - kx^4\right) dt$

(B) $\int_{t_1}^{t_2} \left(\frac{1}{2}mv^2\right) dt$

(C) $\int_{t_1}^{t_2} (mxv)\, dt$

(D) $\int_{x_1}^{x_2} \left(\frac{1}{2}mv^2 + kx^4\right) dx$

(E) $\int_{x_1}^{x_2} (mv)\, dx$