

Waves and optics, solutions

Common themes

Theme 1: Polarizers

Theme 2: Diffraction and interference

Theme 3: Mirrors and thin lenses

Problem 1:

(D) **Optical instruments**

Keplerian telescope: separation: $f_1 + f_2$, magnification: f_1/f_2 . (theme 3)

Problem 2:

(B) **Polarizers**

transmitted intensity/incident intensity = $0.5 \cdot \cos^2(\theta) \cdot \cos^2(90^\circ - \theta)$.

Maximum at $\theta = 45^\circ$. (theme 1)

Problem 3:

(B) **Interference patterns**

Holograms record an interference pattern. A reference beam interferes with a beam transmitted through or reflected by the object. Interference patterns contain amplitude and phase information. (theme 2)

Problem 4:

(E) **Thin film-interference**

When a light wave reflects from a medium with a larger index of refraction, then the phase shift of the reflected wave with respect to the incident wave is π (180°). When a light wave reflects from a medium with a smaller index of refraction, then the phase shift of the reflected wave with respect to the incident wave is zero. (theme 2)

Problem 5:

(A) **Diffraction limited optics**

Light rays from a distant source arrive nearly parallel. Neglecting diffraction, the image of a point will be a disk of approximate diameter d . We want the diameter of the Airy disk (bright center of the diffraction pattern) to be approximately d . The first minimum of the diffraction pattern occurs at approximately $d \sin \theta = \lambda$. We then have $\sin \theta \sim \lambda/d$. Therefore $d \sim (\lambda D)^{1/2}$. (theme 1)

Problem 6:

(E) **Phase and group velocity**

phase velocity: ω/k , group velocity: $d\omega/dk$.

Information (energy) moves with a speed equal to the group velocity.

Problem 7:

(E) **Polarization**

For plane polarization the y- and z-components must be in phase or 180° out of phase. (theme 1)

Problem 8:(D) **Diffraction and interference patterns**

Single slit minimum falls on double slit maximum.

Single slit minimum: $w\sin\theta = m\lambda$.Double slit maxima: $d\sin\theta = n\lambda$. $w/d = m/n$, $md = nw$. Since $d > w$, the only possible answer is D. (theme 2)**Problem 9:**(C) **Mirrors and thin lenses**mirror equation: $1/x_o + 1/x_i = 1/f$, negative image distance \rightarrow virtual image. (theme 3)**Problem 10:**(E) **Diffraction and interference patterns** (theme 2)**Problem 11:**(B) **Diffraction and interference patterns**Double slit interference: $d\sin\theta = \lambda$, $d = 0.01/2000$ m.Small angles: $\theta \sim \lambda/d = 0.1$ rad. 2π rad = 360 deg, 1 rad ~ 60 deg, 0.1 rad ~ 6 deg. (theme 2)**Problem 12:**(D) **The speed of light in a dielectric**

$$c^2 = 1/(\epsilon_0\mu_0), v^2 = 1/(\epsilon\mu), v^2 = c^2/(\kappa_e\kappa_m)$$

Problem 13:(E) **Traveling waves:** $y = A\sin(\omega t - kx)$ represents a wave traveling into the positive x-direction with speed

$$v = \omega/k = \lambda/T$$

Problem 14:(A) **Thin film-interference**The reflections from the front and rear surface of the layer both produce a phase shift of π .Traversing the layer twice (back and forth) also must introduce a phase shift of π . The waves reflected from the front and back surface of the layer must be out of phase by $1/2$ wavelength to destructively interfere. (theme 2)**Problem 15:**(C) **Total internal reflection**

$$\sin\theta_c = n_{\text{small}}/n_{\text{big}} = 1/1.33 = 3/4, \theta_c \sim 50^\circ.$$