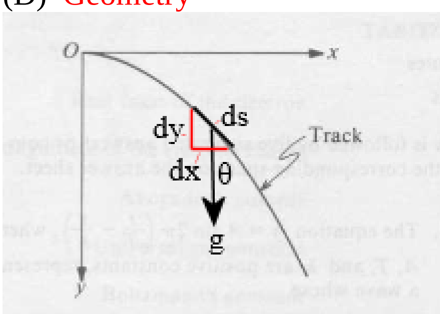
Solutions

Common themes:
Theme 1: relationships between potential energy, kinetic energy and speed
Theme 2: Kepler's laws

Problem 1:
(A) Momentum Conservation
This is an inelastic collision. Momentum is conserved, but kinetic energy changes.
$v_i \propto \sqrt{h_0}$, $v_f = v_i/4$, $v_f = \propto\sqrt{h}$. (theme 1)

Problem 2:
(B) Newton's second law
$\mathbf{a} = \mathbf{g} - \alpha\mathbf{v}$. Only at the top is $\mathbf{v} = 0$.

Problem 3:
(D) Geometry


The component of **g** tangent to the curve is g cosθ.
$\cos\theta = dy/ds$, $g(dy/ds) = g\,dy/(dy^2 + dx^2)^{1/2}$
$= g(x\,dx/2)/((x^2/4 + 1)dx^2)^{1/2} = g(x/2)/(x^2/4 + 1)^{1/2} = gx/(x^2 + 4)^{1/2}$.

Problem 4:
(A) The Kepler problem
What are the possible orbits in an attractive 1/r potential? (theme 2)

Problem 5:
(E) The center of mass
$x_{CM} = (\int_0^L \rho x\,dx)/(\int_0^L \rho\,dx) = (\int_0^L x^2 dx)/(\int_0^L x\,dx) = 2L/3$.
or: What is the only possible answer given that the density increases with x?

Problem 6:
(C) Angular momentum conservation
Angular momentum of interacting objects is conserved, but angular position is not.

Problem 7:
(E) Momentum conservation, Work-kinetic energy theorem:
$\frac{1}{2}(m + M)V^2 = \mu_k(m + M)gs$ $s = V^2/(2\mu_k g) = v^2/(2\mu_k g)(m/(m + M))^2$. (theme 1)

Problem 8:
(E) The harmonic oscillator
$bx_{max}^2/2 = ½ mv^2$, $x_{max} = 1$. (theme 1)

Problem 9:
(E) Newton's law of gravitation, centripetal acceleration
We need $GMm/r^2 > m\omega^2 r$.
$\omega < (GM/r^3)^{1/2} = (4\pi\rho G/3)$, $= 2\pi/\omega > (3\pi/(\rho G))^{1/2}$.

Problem 10:
(C) Gravitational potential energy

Problem 11:
(D) Rotational kinetic energy

$$\Delta T = \frac{1}{2} I\left(\omega_0{}^2 - \omega'^2\right) = \frac{1}{2} \times 4 \times \left(80^2 - 40^2\right) = 9600\text{J}$$

Problem 12:
(C) Newton's second law, $\mathbf{F} = d\mathbf{p}/dt$
$\Delta P = \int F dt$ = area under curve. $\Delta P = 12$ Ns.

Problem 13:
(A) Kepler's third law
$(T_2/T_1)^2 = (R_2/R_1)^3 = (1.01)^3 = (1 + 0.01)^3 \sim 1 + 0.03$.
$T_2/T_1 = (1 + x)^{1/2} \sim 1 + ½x = 1.015$. (theme 2)

Problem 14:
(A) Equilibrium, $\mathbf{F}_{net} = 0$
Let T = the tension in the string. $T\cos\theta = mg = 20$N, $T\sin\theta = 10$ N, $\tan\theta = 0.5$.

Problem 15:
(D) The coefficient of restitution
Let E_i be the energy after the ith bounce. $E_1/E_0 = E_{i+1}/E_i = H/H_0$.
$v_{i+1}/v_i = \text{sqrt}(H/H_0) = (0.8)^{1/2}$, $v_{n-1} = v_0(0.8)^{(N-1)/2}$.
$v_0 = (2gH_0)^{1/2}$. (theme 1)